Trade, Diffusion and the Gains from Openness

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Abstract

Building on Eaton and Kortum's (2002) model of Ricardian trade, Alvarez and Lucas (2005) calculate that a small country representing 1% of the world's GDP experiences a gain of 41% as it goes from autarky to frictionless trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. This paper presents and then calibrates a model where countries interact through trade as well as diffusion of ideas, and then quantifies the overall gains from openness and the role of trade in generating these gains. Having the model match the trade data (i.e., the gravity equation) and the observed growth rate is critical for this quantification to be reasonable. The main result of the paper is that, compared to the model without diffusion, the gains from openness are much larger $(206\% - 240\%)$ and the gains from trade are smaller $(13\% - 24\%)$ when diffusion is included in the model. This last result is a consequence of a novel feature of the model, namely that trade and diffusion behave as substitutes, implying that trade generates smaller gains when diffusion is present.

1   Introduction

How much does a country gain from its relationship with the rest of the world? Consider for example the recent work by Alvarez and Lucas (2005), who build on Eaton and Kortum's (2002) model of Ricardian trade. According to their quantitative model, a small country like Argentina, which represents approximately 1% of the world's GDP, experiences an income gain of 41% as it goes from autarky to *frictionless* trade with the rest of the world. But the gains from *openness*, which includes not only trade but all the other ways through which countries interact, are arguably much higher than the gains from trade. Even if a country were to shut down trade, it could still benefit from foreign ideas through foreign direct investment (FDI), migration, books, journals, the Internet, etc.

The goal of this paper is to construct and calibrate a model where countries interact through trade and diffusion of ideas, and then to quantify the overall gains from openness and the role of trade in generating these gains. The main result is that the gains from trade are smaller than those quantified by Alvarez and Lucas (between 13% and 24% rather than 41% for a country with 1% of the world's GDP) whereas the gains from openness are relatively large (between 206% and 240% for a country with 1% of the world's GDP). An implication is that shutting down trade would generate loses that are quite small in comparison to the loses that would arise if the country were to become completely isolated by shutting down *both* trade and diffusion.

Calculating the gains from trade in a model that allows for trade and diffusion represents a significant departure from the standard practice in the literature, which is to consider trade as the only means through which countries interact. This alternative approach has at least two advantages. First, having both trade and diffusion in the model shows that the gains from trade depend on the way in which trade and diffusion interact. In the model I present here, trade and diffusion are both substitutes and complements. They are substitutes in that if a country cannot import a good then it may adopt a foreign technology for domestic production, and if a country cannot use a foreign technology then it may import the goods produced abroad with that technology.[1] They are complements in that stronger diffusion of ideas from rich to poor countries increases the share of goods that will be produced in the later countries, expanding trade. Indeed, the tremendous expansion of exports from China over recent decades can be seen

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[1] This result is similar to the substitutive property between trade and factor flows (Mundell, 1957), or trade and multinational production (see Helpman, Melitz and Yeaple (2004) and Head and Riess (2004) for recent treatments).

as the result of this country benefiting from increased transfers of technology from rich countries. In the calibrated model, and focusing on the implications of trade and diffusion for advanced countries, the first channel (substitution) dominates the second one (complementarity), so trade and diffusion behave as substitutes. This implies that shutting down trade in this model leads to smaller losses than in models with no diffusion such as Eaton and Kortum (2002) and Alvarez and Lucas (2005).

A second advantage from studying diffusion and trade together is that one can compare the gains from trade with the overall gains from openness, and this may provide a way to judge whether the numbers are reasonable. The usual reaction of economists to the calculated gains from trade in quantitative models is that they are "too small." Apparently, economists have a prior belief that these gains are much higher, so there has been a search for mechanisms through which trade can have a larger effect, such as scale effects, intra-industry reallocations or gains from increased variety. But the result of this search has generally been disappointing (see Tybout, 2003). This paper suggests that the reason for this may be that the gains from trade are in fact "small," while economists' priors about large gains may in fact be about the overall gains from openness. More importantly, this strategy may have relevant implications for research and policy regarding how countries integrate with the rest of the world. In particular, the result of this paper that the gains from trade appear to be quite small relative to the overall gains from openness suggests that both research and policy should at least partially redirect their attention from trade to all the other ways through which countries interact. More attention should be devoted, for example, to understanding the importance of FDI and migration in the international exchange of ideas, and to think about policies that countries can follow to speed up the adoption of foreign technologies.

In Eaton and Kortum's (2002) model of Ricardian trade with no diffusion, countries gain from openness through specialization according to comparative advantage. In the model I construct here, countries also gain from diffusion of ideas. Both the gains from trade and the gains from diffusion come from the same basic phenomenon, namely the sharing of the best ideas across countries. Consider, for example, Japan's superior technology for producing automobiles. This technology can be shared through trade by having Japan export automobiles or through diffusion by having other countries produce their own automobiles using Japan's technology. In both cases, thanks to the non-rivalry of ideas emphasized by Romer (1990), sharing ideas leads to an increase in worldwide income.

These gains from sharing ideas are the same ones that give rise to aggregate increasing returns to scale in models of quasi-endogenous growth such as Jones (1995) and Kortum (1997). Consider Kortum (1997). In the simplest version of this model, the arrival of new ideas is proportional to the population level and the quality of each idea is drawn from an unchanging distribution. The technology frontier at a certain point in time is the set of best ideas available to produce the given set of goods, and the average productivity of the technology frontier determines the per-capita income level. A larger economy has more ideas, more ideas imply that the best available technologies are more productive, and this allows the economy to sustain a higher income level. This entails a scale effect in *levels* so that income per capita y is increasing with population L, $y = \beta L^{\eta}$, where β and η are positive constants. Jared Diamond's main argument in his book *Guns, Germs and Steel* can be interpreted as saying that this scale effect from sharing ideas is what allowed large "Eurasia" to attain a superior level of productivity (Diamond, 1997). For the present purposes, the relevant implication is that a country can achieve a level of income that is much lower in isolation than sharing ideas with the rest of the world.

A scale effect of the kind just described is the key element in quasi-endogenous growth models, as it implies that the growth rate is proportional to the growth rate of population, $g = \eta g_L$. This implication allows for a simple calibration, which reveals the magnitude of the gains from openness (in steady state levels). With $g = 1.5\%$ and $g_L = 4.8\%$,[2] the equation $g = \eta g_L$ implies that $\eta = 0.31$, which in turn implies that a country with 1% of the world's population enjoys gains from openness equal to 320% ($100^{0.31} = 4.2$).

To quantify the gains from openness and explore the role of trade in generating these gains, it is necessary to have a model that is quantitatively consistent with both the observed growth rate and the observed trade volumes. I build on Eaton and Kortum's (2001) model of trade and growth, which can be seen as an extension of Kortum (1997) to incorporate trade. A key parameter in this model, θ, determines the variability of the distribution of the quality of ideas.[3]

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[2] The rate of growth of y is the rate of growth of income per worker after subtracting the contribution from increases in average human capital and in the capital-output ratio (see Jones, 2002, and Klenow and Rodríguez-Clare, 2005). The value for g_L comes from the rate of growth of researchers in the G5 countries (West Germany, France, the United Kingdom, the United States, and Japan) from 1950 to 1993, see Jones (2002). Note that this is significantly higher than the 1.1% rate of growth of population observed in the OECD in the last decades because of an increasing share of the population devoted to research. Doing this exercise with a lower g_L would lead to even larger gains from openness.

[3] In Eaton and Kortum (2001) the quality of ideas is distributed Pareto with parameter θ. Thus, the variance of this distribution *increases* as θ *falls*. I instead follow Alvarez and Lucas (2005), who flip this parameter

If θ is calibrated to match the gravity equation, as is done in Eaton and Kortum (2002), then a puzzle emerges in that the implied growth rate is almost an order of magnitude lower than the one we observe for the OECD countries in the last decades. Alternatively, if θ is calibrated to match the observed rate of growth in the OECD, then the model generates too much trade, since the pattern of comparative advantage is too strong and dominates the estimated trade costs.

One way to deal with this puzzle is by allowing for diffusion of ideas across countries.[4] To understand why diffusion makes it possible to match both the gravity equation and the growth rate, note that the excessive volume of trade generated by the high θ needed to match growth of 1.5% per year is dampened when countries can share ideas through diffusion rather than trade. Introducing diffusion into the model leads to a gravity equation with a discontinuous border effect (i.e., trade falls discontinuously as trade costs increase from zero) that is not present in Eaton and Kortum (2001, 2002). Estimating θ from this equation leads to $\theta = 0.22$ rather than Eaton and Kortum's $\theta = 0.12$, and this helps to increase the model's implied growth rate from $g = 0.29\%$ to $g = 0.53\%$. But this is still significantly below the observed $g = 1.5\%$.

To increase the model's implied growth rate without affecting its trade implications, I allow for progress and diffusion in ideas that are relevant for non-tradable goods. I refer to these ideas as "NT ideas" to differentiate them from the ideas associated with tradable goods, which I will call "T ideas." Analogously to the role played by θ for T ideas, a parameter γ determines the variability of the distribution of the quality of NT ideas. One can then use $\theta = 0.22$ to match the gravity equation, and $\gamma = 0.2$ so that the model generates $g = 1.5\%$.[5] Having this model that is quantitatively consistent with observed growth and trade volumes, I can then calculate the gains from openness and the role of trade in these gains. The main result is as stated above: the gains from openness are large $(206\% - 240\%)$, while the gains from trade are in fact smaller than in the model without diffusion $(13\% - 24\%$ rather than $41\%)$.[6] These results are

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around and have a higher θ increase the variability of the quality of ideas.

[4] An alternative approach is to allow for knowledge spillovers as a way to accelerate the rate of growth of ideas (I thank Sam Kortum for suggesting this possibility). In a previous version of this paper I explored a model with such spillovers and calculated the corresponding gains from openness and the role of trade. The results are very similar to the ones I present below.

[5] In the calibrated model the growth rate is $g = (\theta/2 + \gamma)g_L$. Thus, $\theta = 0.22$ and $\gamma = 0.2$ together with $g_L = 4.8\%$ imply $g = 1.5\%$.

[6] These gains from openness differ from the ones calculated above for the simple calibration to observed growth (i.e., 320%) because the model developed in the paper and its calibration incorporate frictions in the diffusion process that lower the gains from openness.

derived for the case in which there are no trade costs, so these computed gains are an upper bound of the actual gains. An alternative calibration allows for such costs and computes the gains from openness and trade for a set of 19 OECD countries. The results imply that Finland, which accounts for roughly 1% of the world's research in the calibrated model, has gains from openness of 174% and gains from trade of 9%. The average of the corresponding gains for the 19 countries considered are 143% and 9%.

This paper is related to the literature on trade and endogenous growth associated with Grossman and Helpman (1991) and Rivera-Batiz and Romer (1991), among others. This group of papers showed that trade or international knowledge spillovers could lead to a higher growth rate thanks to the exploitation of scale economies in R&D at the global level. This is essentially what Jones (1995) called a "strong scale effect," whereby larger markets exhibit higher growth rates. Jones' empirical analysis showed that such a strong scale effect is not consistent with the data, however, so there has been a shift towards quasi-endogenous growth models, where the growth rate is not affected by scale variables. In this paper I focus on this class of models and explore the *quantitative* implications of openness on steady state income levels.

Another related literature is the one that focuses on international technology diffusion. The closest paper is by Eaton and Kortum (1999), who develop and calibrate a model of technology diffusion and growth among the five leading research economies. These authors then perform a counterfactual analysis to see the implications for the U.S. of detaching itself from sharing ideas with the rest of the world. Using the quasi-endogenous growth model due to Jones (1995), Klenow and Rodríguez-Clare (2005) performed a similar exercise and found enormous gains from openness for small countries. This paper can be seen as an extension of this literature to include trade into the model and thereby quantify the gains from openness arising from both trade and diffusion.

Finally, Coe and Helpman (1995), Keller (1998) and others reviewed in Keller (2004) study the role of trade as a vehicle for "international R&D spillovers." The idea is that by importing intermediate and capital goods, a country benefits from the R&D done in the exporting countries. This is a key feature of the model of R&D and trade in Eaton and Kortum (2001) as well as the model I present in this paper. But here such R&D spillovers are interpreted as gains from trade, whereas technology diffusion is a term reserved for the more narrow concept of information flows that allow countries to *directly* use technologies created elsewhere. In other words, the gains from international R&D spillovers in Coe and Helpman (1995) are here simply

measured as gains from trade. A different notion is that trade accelerates the international flow of technical know-how (see Grossman and Helpman, p. 165). Several papers have explored this empirically with mixed results (see Rhee et. al., 1984, Aitken et. al., 1997, and Clerides et. al., 1998, and Bernard and Jensen, 1999). This phenomenon is not captured in the model presented below.

The rest of the paper is organized as follows. In the next section I lay out the basic model with T ideas and no diffusion to introduce the basic notation and assumptions, and to establish a benchmark against which to compare the results of the full model. In this section I also show that if θ is calibrated to match trade volumes then the implied growth rate is too low. In Section 3 I present the full model, which builds on the model of Section 2 by adding both technological progress in the production of non-tradables through the introduction of NT ideas, and diffusion for both T ideas and NT ideas. In this section I derive analytical results for the gains from openness and the gains from frictionless trade. I establish the result discussed above that trade and diffusion are substitutes, and show that this implies that the gains from trade are lower than in a model with no diffusion. In section 4 I calibrate the model to match trade volumes and the observed growth rate, and in Section 5 I use the calibrated model to quantify the gains from openness and the role of trade. Section 6 explores the impact of transportation costs on the gains from trade and openness. The final section offers concluding comments and topics for future research.

2 Trade and growth without diffusion

In this section I first present a model of trade and growth without diffusion based on Eaton and Kortum (2001). I then calibrate an enriched version of the model to compute the gains from trade and the implied growth rate.

2.1 A model of trade and growth

There is a single factor of production, labor, I countries indexed by i, and a continuum of tradable intermediate goods indexed by $u \in [0, 1]$. The intermediate goods are used to produce a final consumption good via a CES production function with an elasticity of substitution $\sigma > 0$. The productivity with which individual intermediate goods are produced (i.e., output per unit of the labor) varies across intermediate goods u and across countries, and this gives rise to

trade. Let us focus on a single country for now so that we can momentarily leave aside the use of country subscripts. It is convenient to work with the inverse of productivity. To do so, let $x(u)$ be a parameter that determines the cost of producing intermediate good u. In particular, let the cost of producing such a good be given by $x(u)^\theta w$, where w is the wage level. Note that the parameter θ, which will be constant across goods and countries, magnifies the variability of the cost parameter x on the actual cost structure across goods and countries. This parameter will be crucial in the analysis that follows.

At any point in time the cost parameters $x(u)$ are the result of previous research efforts in each country. Following Kortum (1997) and Eaton and Kortum (2001), research is modeled as the creation of ideas, although for simplicity here I assume that this is exogenous. In particular, I assume that there is an instantaneous (and constant) rate of arrival ϕ of new ideas per person. In the concluding section I argue that the main results of the paper should not change significantly if research efforts were endogenous.

Ideas are specific to goods, and the good to which an idea applies is drawn from a uniform distribution in $u \in [0, 1]$. Since this interval has unitary mass, then at time t there is a probability $R(t) \equiv \phi L(t)$ of drawing an idea for any particular good, where $L(t)$ is the population level at time t. This implies that the arrival of ideas is a Poisson process with rate function $\phi L(t)$, so the number of ideas that have arrived for a particular good by time t is distributed Poisson with rate $\lambda(t) \equiv \int_0^t R(s)ds$. Again, since the set of goods has unitary mass, then $\lambda(t)$ also represents the total stock of ideas (applying to all goods) at time t. (From here onwards, I will suppress the time index as long as it does not cause confusion.) Assuming that L grows at the constant rate g_L (assumed to be common across countries) then in steady state we must have $\lambda = R/g_L$, so λ also grows at rate g_L.

Ideas for producing a particular intermediate good differ only in terms of a "quality" parameter, and the economy's productivity for intermediate good u is determined by the best idea available for the production of this good. The quality of ideas is independently drawn from a distribution of quality which is assumed to be Pareto with support in $[1, \infty]$ and parameter one.[7,8] Letting $x(u)$ be the inverse of the quality of the best idea that has arrived up to time t

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[7]Kortum (1997) shows that the Pareto assumption for the distribution of quality is necessary for there to be a steady state growth path.

[8]Eaton and Kortum (2001) assume that the distribution of quality is Pareto with parameter θ, whereas here I assume instead a Pareto distribution with parameter 1, with θ being a parameter that expands the cost differences across ideas, as in Alvarez and Lucas (2005). The two approaches are equivalent except that the θ here is the inverse of Eaton and Kortum's θ.

for good u, then it is easy to show that $x(u)$ is distributed exponentially with parameter λ.[9]

Transportation costs are of the iceberg type, with one unit of a good shipped from country j resulting in $k_{ij} \leq 1$ units arriving in country i. I assume that $k_{ii} = 1$, that $k_{ij} = k_{ji}$, and that the triangular inequality holds (i.e., $k_{ij} \geq k_{il}k_{lj}$ for all i, j, l).

2.1.1 Equilibrium

Following Alvarez and Lucas (2005), I relabel goods by $x \equiv (x_1, x_2...x_I)$ rather than u. The price of good x in country i is

$$p_i(x) = \min_j \left\{ \left(\frac{w_j}{k_{ij}} \right) x_j^\theta \right\}$$

Letting $s_i(x) \equiv \min_j \left\{ (w_j/k_{ij})^{1/\theta} x_j \right\}$, then $p_i(x) = s_i(x)^\theta$. From the properties of the exponential distribution it follows that $s_i(x)$ is distributed exponentially with parameter ψ_i,[10] where

$$\psi_i \equiv \sum_j \psi_{ij} \text{ and } \psi_{ij} \equiv (w_j/k_{ij})^{-1/\theta} \lambda_j \tag{1}$$

Letting p_{mi} be the price index of the final good, then $p_{mi}^{1-\sigma} = \int p_i(x)^{1-\sigma} dF(x)$ and assuming $1 + \theta(1 - \sigma) > 0$,[11] we get

$$p_{mi} = C_T \psi_i^{-\theta} \tag{2}$$

where $C_T = \Gamma[1 + \theta(1 - \sigma)]^{1/(1-\sigma)}$, with $\Gamma()$ being the Gamma function.

To determine wages we introduce the trade-balance conditions. As shown by Eaton and Kortum (2002), the average price charged by any country j in any country i is the same, and hence the share of total income in country i spent on imports from country j, D_{ij}, is equal to the share of goods for which country j is the lowest cost supplier in country i. In turn, this share is equal to the probability that $(w_j/k_{ij})x_j^\theta = \min_l\{(w_l/k_{il})x_l^\theta\}$. From the properties of the

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[9]Letting q represent the quality of ideas, then $\Pr(Q \leq q) = H(q) = 1 - 1/q$. Letting v be the quality of the best idea that has arrived up to time t, then using $e^x \equiv \sum_{k=0}^\infty x^k/k!$ we get $\Pr(V \leq v) = \sum_{k=0}^\infty \left(e^{-\lambda}(\lambda)^k/k! \right) H(v)^k = e^{-\lambda/v}$, and hence $x \equiv 1/v \sim \exp(\lambda)$. There is a discrepancy in that here $v \geq 1$ (because $q \geq 1$) whereas the exponential distribution has range in $[0, \infty[$. As shown by Kortum (1997), this can be safely ignored because quality levels below one become irrelevant as λ gets large.

[10]These properties are: (1) if $x \sim \exp(\lambda)$ and $k > 0$ then $kx \sim \exp(\lambda/k)$; and (2) if x and y are independent, $x \sim \exp(\lambda)$ and $y \sim \exp(\mu)$, then $\min\{x, y\} \sim \exp(\lambda + \mu)$.

[11]The assumption that $1 + \theta(1 - \sigma) > 0$ entails $\sigma < 1 + 1/\theta$. In principle, I could explore whether this inequality holds given estimates of σ and given the values of θ that I will discuss in the text below. In practice, however, the empirical value of σ depends on the level of aggregation that we use for inputs, which in turn should be determined by the level at which technologies differ in the way specified in the model. Thus, the restriction $1 + \theta(1 - \sigma) > 0$ must be taken as an assumption for now.

exponential distribution, this probability is $D_{ij} \equiv \psi_{ij}/\psi_i$. Given that total income in country i is $L_i w_i$, then the trade balance conditions are simply

$$L_i w_i = \sum_j L_j w_j D_{ji} \qquad (3)$$

The previous conditions determine a competitive equilibrium. In particular, a competitive equilibrium at any point in time is a couple of vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mI})$ and $w = (w_1, w_2, ..., w_I)$ such that, together with the vector $(\psi_1, \psi_2, ..., \psi_I)$ that satisfies equations (1) and (2) and $D_{ij} \equiv \psi_{ij}/\psi_i$, the trade balance conditions (3) are satisfied.

2.1.2 Growth and the gains from trade

I now turn to the implications of the model for growth and the gains from trade. Once we choose a numeraire, wages are constant in steady state since all λ_i are growing at the same rate g_L. The growth rate in real wages is then given by the rate of decline in p_{mi}. But from (2) it is clear that p_{mi} falls at rate θg_L, so the growth rate of real wages or consumption is

$$g = \theta g_L \qquad (4)$$

This is a simple version of Kortum (1997). Note in particular that growth of income per capita depends on the growth rate of population (the hallmark of quasi-endogenous growth models) and that a higher θ implies a higher growth rate. The reason for this positive role of θ is that a high θ magnifies the benefit of high-quality ideas and this is the mechanism that fuels growth in this model.

The gains from trade are determined by the increase in the real wage, w_i/p_{mi}, as a country goes from autarky to trade. In calculating these gains here and in the following sections I primarily focus on the case of frictionless trade because this allows for simpler derivations and because this establishes an upper bound for the gains from trade. In autarky $\psi_i = w_i^{1/\theta} \lambda_i$. Plugging into (2) and using $\lambda_i = R_i/g_L$ yields $w_i/p_{mi} = C_T^{-1}(R_i/g_L)^\theta$. Similarly, with frictionless trade we have $p_m = C_T \left(\sum_j w_j^{-1/\theta} R_j/g_L \right)^{-\theta}$. Assuming $\phi_i = \phi$, it is easy to show that there is factor price equalization (i.e., $w_i = w_j$ for all i, j), and hence the gains from trade are simply

$$GT_i = \left(\frac{\sum L_i}{L_i} \right)^\theta$$

Since they generate a smaller share of the world's best ideas, smaller economies have more to gain from integrating with the rest of the world. Moreover, a high θ leads to higher gains

from trade. As explained in the Introduction, the reason for this is that a high θ increases the variability of cost differences across countries and hence leads to a stronger pattern of comparative advantage.
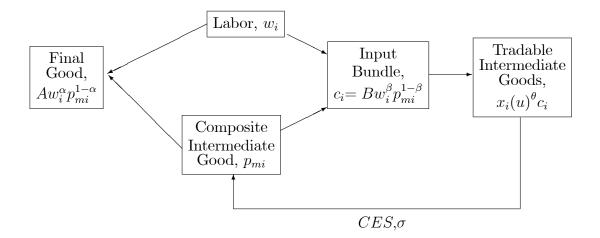
2.2 Towards a quantitative model

I now enrich and calibrate the model to explore its quantitative implications. Following Eaton and Kortum (2002) and Alvarez and Lucas (2005), I introduce two modifications. First, it is assumed that intermediate goods are used in the production of intermediate goods, thus generating a "multiplier" effect that expands the gains from trade and the growth rate. Second, it is assumed that production of the consumption good uses labor directly and not only through intermediate goods. This is done to capture the existence of non-tradables that dampen the gains from trade. In the model of this section (but not in the one of Section 3) this will also reduce the growth rate because technological progress is confined to tradable intermediates.

These two modifications are illustrated in Figure 1 and captured formally as follows. The intermediate goods are used to produce a "composite intermediate good" with a CES production function with elasticity σ, so that p_{mi} - which above was the price index of the consumption good - is now the price index of this composite good. In turn, the composite good together with labor are used to produce intermediate goods with a Cobb-Douglas production function with labor share β. One can think of an "input bundle" produced from labor and the composite intermediate good that is in turn used to produce all the intermediate goods. The cost of the input bundle in country i is then $c_i \equiv B w_i^\beta p_{mi}^{1-\beta}$ where $B \equiv \beta^{-\beta} (1-\beta)^{\beta-1}$, while the cost of intermediate good u in country i is now $x_i(u)^\theta c_i$. Finally, the consumption good is produced from the composite intermediate good and labor with a Cobb-Douglas technology with labor share α. Thus, the price of the consumption good is $p_i = A w_i^\alpha p_{mi}^{1-\alpha}$, where $A \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$. Note that if $\beta = 1$ and $\alpha = 0$ then we are back to the model above. The individual intermediate goods are the only tradeable goods.

These modifications do not substantially affect the qualitative results above; the only difference is that now the wage w_i must be substituted by the unit cost of the input bundle, c_i, in the definition of ψ_{ij} in equation (1).[12] But there are important quantitative implications. In

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[12] As shown by Alvarez and Lucas (2005), the trade balance conditions are not affected by the values of α or β (at least for the case in which there are no tariffs, as here).

Figure 1: The Production Structure

particular, the growth rate is now

$$g = \theta \left(\frac{1 - \alpha}{\beta} \right) g_L \tag{5}$$

If intermediate goods have a high share in the production of intermediate goods (i.e., high $1-\beta$), so that there is a large multiplier $1/\beta$, then the growth rate will be higher. Similarly, the growth rate increases with the share of intermediate goods in the production of the consumption good (i.e., $1 - \alpha$). The term $\left(\frac{1-\alpha}{\beta} \right)$ also affects the gains from trade, which in the case of $\phi_i = \phi$ considered above are

$$GT_i = \left(\frac{\sum L_i}{L_i} \right)^{\theta(1-\alpha)/\beta} \tag{6}$$

The key parameters of the model are θ, α, β, and g_L. Eaton and Kortum (2002) estimate θ from the gravity equation generated by the model together with bilateral import and price data for the OECD countries. They focus on the way in which θ determines the impact of trade costs on trade volumes. To isolate this aspect of the gravity equation, Eaton and Kortum focus on "normalized trade flows." Let the normalized bilateral imports of country i from country j be D_{ij}/D_{jj}. If there are no trade costs then $D_{ij}/D_{jj} = 1$ for all i, j. With trade costs we have

$$D_{ij}/D_{jj} = \left(\frac{p_{mj}}{p_{mi}k_{ij}} \right)^{-1/\theta}$$

Taking logs, and letting $m_{ij} \equiv \ln(D_{ij}/D_{jj})$ and $\kappa_{ij} = \ln(p_{mj}/p_{mi}k_{ij})$, then

$$m_{ij} = -(1/\theta)\kappa_{ij} \tag{7}$$

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Eaton and Kortum (2002) construct m_{ij} from 1990 data on trade and production of manufactures for 19 OECD countries and κ_{ij} from data on prices from the UN ICP 1990 benchmark study, which gives retail prices for 50 manufactured products in these countries.[13] An OLS regression with no intercept yields $\theta = 0.12$.[14]

Alvarez and Lucas (2005) calibrate the parameters α and β to match the fraction of U.S. employment in the non-tradables sector and the share of labor in the total value of tradables produced, respectively. They find $\alpha = 0.75$ and $\beta = 0.5$. For g_L I could use the growth rate of population in the OECD over the last decades, which is $g_L = 1.1\%$. But as Jones (2002) has emphasized, there has been an upward trend in the share of people devoted to R&D in rich countries over the last decades. According to Jones, the rate of growth of researchers has been 4.8% over the period 1950-1993 in the G-5 countries.[15] Plugging these values in equation (5) together with Eaton and Kortum's $\theta = 0.12$ yields $g = 0.29\%$, which is significantly lower than the observed rate of growth of productivity in the OECD countries, which is close to $g = 1.5\%$. One could, of course, calibrate θ to match the observed growth rate, but this would lead to inconsistent implications for the role of gravity in trade. In particular, bilateral trade volumes would decline too slowly as trade costs increase.

Turning to the gains from trade, these parameters ($\theta = 0.12$, $\alpha = 0.75$, and $\beta = 0.5$) imply from (6) that the gains from frictionless trade for a country with 1% of the world's total population are $100^{0.06} = 1.3$, or 30%.[16] If instead we use the "central value" of θ in Alvarez and Lucas (2005), namely $\theta = 0.15$, then the gains from trade are 41%, as mentioned in the Introduction.

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[13]The 19 countries included in the sample are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom and the United States.

[14]The OLS estimation yields $1/\theta = 8.03$ with a standard error of 0.15. The R-squared is 0.06. A simple method of moments estimation of $1/\theta$ in (7) yields basically the same outcome (see Eaton and Kortum, 2002).

[15]The G5 countries are France, West Germany, the United Kingdom, the United States, and Japan. Clearly, an increasing share of people engaged in research implies that the system is not in steady state, but the system can still attain a constant growth rate where these formulas are valid (see Jones, 2002).

[16]Note that if $\phi_i = \phi_j$ for all i, j then with frictionless trade wages are equal across countries, so a country with 1% of the population also has 1% of the world's GDP. For convenience, I used this case to refer to the gains from trade in the Introduction.

3   Diffusion, trade and growth

In this section I extend the previous model to introduce international diffusion of ideas and make it quantitatively consistent with the observed growth rate and trade volumes. First, I allow for technological progress in the production of non-tradables. In particular, I assume that just as there are ideas that increase the productivity of tradable intermediate goods, there are ideas that increase the productivity of non-tradable consumption goods. I will refer to the first type of ideas as "T ideas" and to the second type of ideas as "NT ideas." (I will suppress the T and NT labels except when necessary to avoid confusion.) Second, I allow for international diffusion of both types of ideas. The introduction of NT ideas into the model is necessary to have the model match the observed growth rate, whereas diffusion of both T and NT ideas is a key mechanism for the gains from openness that I want to explore. Moreover, as will be shown, diffusion of T ideas helps to make the model better match the trade data.

To model the role of NT ideas, I assume that there is a continuum of non-tradeable consumption goods indexed by $v \in [0, 1]$. These goods enter the representative consumer's instantaneous utility through CES preferences with elasticity of substitution σ.[17] They are produced from labor and the composite intermediate good with a Cobb-Douglas production function at cost $Az(v)^\gamma w^\alpha p_m^{1-\alpha}$, where $z(v)$ is a cost parameter associated with good v. Analogously to the way in which T ideas determine the cost parameters $x(u)$ for intermediate goods, $z(v)$ is the inverse of the quality of the best NT idea that has arrived for good v. Note that the parameter γ plays the same role in affecting the cost of non-tradeable consumption goods as the parameter θ plays in affecting the cost of the tradeable intermediate goods.

The generation and diffusion of T and NT ideas is assumed to be identical, so I suppress the T and NT labels for now. I assume that the world is composed of two regions: the North and the South. To simplify, I take the South to be a single economy, whereas the North contains I countries. I denote the set of north countries by Ω_N and similarly use Ω_S to denote the (unitary) set of South countries. I use index i for north countries and the indexes j and l for all countries (i.e., $i \in \Omega_N$ and $j, l \in \Omega_N \cup \Omega_S$).

Only countries in the North generate ideas. Ideas at first are "national" (as in the previous section), but then diffuse to other north countries, from which they finally diffuse to the South. Thus, there are $I + 2$ pools of ideas: one pool for each north country, a pool of ideas that have

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[17] The assumption that the elasticity of substitution is the same here as in the production of the composite good is made to minimize notation and plays absolutely no role in the results.

diffused among the north countries (the "north ideas"), and the pool of ideas that have diffused to the South. Ideas in the latter pool are available in all countries, so I refer to these ideas as "global ideas."

Following Krugman (1979) and Eaton and Kortum (2006), I assume that diffusion is probabilistic, with each idea having a constant probability of diffusing. Let δ be the rate of diffusion among countries in the North and let δ' be the rate of diffusion from the pool of north ideas to the pool of global ideas. Letting λ_N and λ_G be the stocks of ideas in the north and global pools, respectively, then $\dot{\lambda}_i = \phi_i L_i - \delta \lambda_i$, $\dot{\lambda}_N = \delta \sum \lambda_i - \delta' \lambda_N$, $\dot{\lambda}_G = \delta' \lambda_N$. In steady state the stock of national ideas in north country i is

$$\lambda_i = (\phi_i/(g_L + \delta))L_i \tag{8}$$

while the stock of north ideas is

$$\lambda_N = \widetilde{\delta} \sum \lambda_i \tag{9}$$

where $\widetilde{\delta} \equiv \delta/(\delta' + g_L)$. Finally, the stock of global ideas in steady state is

$$\lambda_G = \delta' \lambda_N / g_L \tag{10}$$

All these stocks of ideas grow at rate g_L in steady state.

3.1   Equilibrium

Let us first focus on consumption goods. Since they are non-tradable then we care only about the best idea available in each country, irrespective of whether they are national ideas or not. That is, for north country i the cost parameter for a consumption good is associated with the best idea across the pools of national ideas in i, north ideas and global ideas. This implies that the cost parameter in north country i for any consumption good is distributed exponentially with parameter $\lambda_i + \lambda_N + \lambda_G$. Similarly, the cost parameter for a consumption good in the South is determined by the best global idea and is distributed exponentially with parameter λ_G. In equilibrium, consumption goods are sold at cost, hence the price of consumption good v is $Az(v)^\gamma w^\alpha p_m^{1-\alpha}$. The price index for the consumption bundle is then

$$p = Aw^\alpha p_m^{1-\alpha} \left(\int_0^1 z(v)^{\gamma(1-\sigma)} dv \right)^{1/(1-\sigma)}$$

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Since $z(v)$ in north country i is distributed exponentially with parameter $\lambda_i + \lambda_N + \lambda_G$, then (assuming that $1 + \gamma(1 - \sigma) > 0$ so that the integral above is well defined) we have that the price index for the consumption bundle in i is

$$p_i = A w_i^\alpha p_{mi}^{1-\alpha} C_{NT} \left(\lambda_i + \lambda_N + \lambda_G\right)^{-\gamma} \tag{11}$$

where $C_{NT} \equiv \Gamma(1 + \gamma(1 - \sigma))^{1/(1-\sigma)}$. The price index for the South is given by a similar expression with $\lambda_i + \lambda_N + \lambda_G$ replaced by λ_G.

Turning to intermediate goods and trade, note that for each good there are n best national ideas (one for each north country), a best north idea, and a best global idea. Recalling that $x_i(u)$ denotes the cost parameter for the best national idea in country i for intermediate good u, and using $x_N(u)$ and $x_G(u)$ to denote the cost parameters associated with the best north and global ideas for this good, respectively, then we can now label intermediate goods by $\widetilde{x} = (x_1, x_2, ..., x_I, x_N, x_G)$. Consider the different ways in which a country l could procure a particular good $\widetilde{x}$. Just as in the previous section, country l could buy this good produced with national ideas from any of the I north countries at minimum cost $\min_i \left\{ (c_i/k_{li}) x_i^\theta \right\}$ (recall that i necessarily belongs to Ω_N). But now it can also buy goods produced with diffused ideas: it can buy goods produced in any of the I north countries with north ideas, and it can buy goods produced in any of these countries plus the South with global ideas. The cost of buying a good produced in country i with the best north idea is $(c_i/k_{li}) x_N^\theta$, so the minimum cost of buying a good produced with a north idea is $\min_i \left\{ (c_i/k_{li}) x_N^\theta \right\}$. Similarly, the minimum cost of buying a good produced with a global idea is $\min_j \left\{ (c_j/k_{lj}) x_G^\theta \right\}$ (note that this minimization now includes the possibility that the good is produced in the South, $j = S$). Letting $\widetilde{c}_l \equiv \min_j \left\{ c_j/k_{lj} \right\}$ and $\widetilde{c}_l^N \equiv \min_i \left\{ c_i/k_{li} \right\}$, then the price of good $\widetilde{x}$ in country l is now

$$p_l(\widetilde{x}) = \min \left\{ \min_i \left\{ (c_i/k_{li}) x_i^\theta \right\}, \widetilde{c}_l^N x_N^\theta, \widetilde{c}_l x_G^\theta \right\} \equiv \xi_l(\widetilde{x})^\theta$$

Given the properties of the exponential distribution, ξ_l is distributed exponentially with parameter

$$\hat{\psi}_l \equiv \sum_i (c_i/k_{li})^{-1/\theta} \lambda_j + \left(\widetilde{c}_l^N\right)^{-1/\theta} \lambda_N + \left(\widetilde{c}_l\right)^{-1/\theta} \lambda_G \tag{12}$$

This parameter determines the price index for intermediate goods in country l. In particular, and analogous to (2), we now have

$$p_{ml} = C_T \hat{\psi}_l^{-\theta} \tag{13}$$

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Wages are determined by the trade-balance conditions, as in (3), but the trade shares are now different. To determine these shares, note that $(c_i/k_{li})^{-1/\theta} \lambda_i /\hat{\psi}_l$ is the share of goods which country l can procure most cheaply from i produced with $i's$ best national ideas. The fact that l may also buy north and global goods from i establishes that

$$D_{li} \geq (c_i/k_{li})^{-1/\theta} \lambda_i /\hat{\psi}_l \text{ for } i \in \Omega_N \tag{14}$$

To proceed, let $M_l^N \equiv \arg\min_i \{c_i/k_{li}\}$ denote the set of countries from which country l would buy all goods produced with north ideas (i.e., if country l buys a good produced with a north idea, it must be buying this good from $i \in M_l^N$). Obviously, $c_i/k_{li} = \tilde{c}_l^N$ if $i \in M_l^N$. The share of goods that country l will actually buy from countries $i \in M_l^N$ produced with north ideas is then given by $\left(\tilde{c}_l^N\right)^{-1/\theta} \lambda_N/\hat{\psi}_l$. The fact that l may also buy global goods from countries $i \in M_l^N$ establishes that

$$\sum_{i \in M_l^N} D_{li} \geq \left(\left(\tilde{c}_l^N\right)^{-1/\theta} /\hat{\psi}_l\right) \left(\sum_{i \in M_l^N} \lambda_i + \lambda_N\right) \tag{15}$$

Finally, let $M_l \equiv \arg\min_j \{c_j/k_{lj}\}$ denote the set of countries from which l would buy all goods produced with global ideas. If the South were the unique member of M_l then country l would buy all goods produced with global ideas from the South, and then

$$D_{lS} = (c_S/k_{lS})^{-1/\theta} \lambda_G/\hat{\psi}_l$$

In this case (15) would have to be satisfied with equality. If there are north countries in M_l, however, then country l will import from these countries goods produced with national, north and global ideas, and hence

$$\sum_{j \in M_l} D_{lj} = \left(\left(\tilde{c}_l\right)^{-1/\theta} /\hat{\psi}_l\right) \left(\sum_{i \in M_l \cap \Omega_N} \lambda_i + \chi \left(M_l \cap \Omega_N\right) \lambda_N + \lambda_G\right) \tag{16}$$

where $\chi \left(M_l \cap \Omega_N\right) = 1$ if $M_l \cap \Omega_N \neq \varnothing$ and $\chi \left(M_l \cap \Omega_N\right) = 0$ otherwise.

The competitive equilibrium is determined by the vectors $p_m = (p_{m1}, p_{m2}, ..., p_{mI}, p_{mS})$ and $w = (w_1, w_2, ..., w_I, w_S)$ such that together with the vector $(\hat{\psi}_1, \hat{\psi}_2, ..., \hat{\psi}_I, \hat{\psi}_S)$ that satisfies equations (12) and (13) and the matrix $\{D_{jl}; j, l = 1, 2..., I, S\}$ that satisfies $(14) - (16)$, the trade-balance conditions (3) are satisfied.

In steady state wages are constant, so the common growth rate is given by $g = -\dot{p}_l/p_l$. But equations (12) and (13) imply that p_{ml} declines at rate $\theta g_L/\beta$ (using $c_l = B w_l^\beta p_{ml}^{1-\beta}$), so from (11) we find

$$g = \left[\theta \left(\frac{1-\alpha}{\beta} \right) + \gamma \right] g_L \tag{17}$$

The growth rate is composed of two terms: the first term, $\theta(\frac{1-\alpha}{\beta})g_L$, is associated with technological progress in tradeable (intermediate) goods, whereas the second term, γg_L, is associated with technological progress in non-tradeable (consumption) goods. It is worth noting that the first term is exactly the same as in the model with no diffusion of the previous section (see equation (5)). This reveals that diffusion has no effect on steady state growth in this model; as will become clear below, there is only a level effect.

3.2 Gains from trade and diffusion

I now turn to the derivation of the gains from trade and diffusion of both T and NT ideas. As in the previous section, I consider the gains from *frictionless* trade. To do so, I first derive the real wage for the case of no trade (with and without diffusion), and then for the case of frictionless trade with diffusion. I then compare these wages to establish the gains from trade and diffusion, and discuss several implications from these results.

3.2.1 No trade

When there is no trade, the price index of intermediate goods in country l, p_{ml}, is given by (13) but with

$$\hat{\psi}_l = c_l^{-1/\theta} \eta_l$$

where η_l is the stock of ideas in country l. For each north country i, this stock is composed of ideas originated in i and foreign ideas that have diffused (i.e., foreign ideas that have become north or global ideas), while for the South this stock is composed entirely of global ideas. Thus,

$$\eta_l = \begin{cases} (1 + \delta/g_L)\lambda_l & \text{if no diffusion and } l \in \Omega_N \\ 0 & \text{if no diffusion and } l \in \Omega_S \\ \lambda_l + \lambda_N + \lambda_G & \text{if there is diffusion and } l \in \Omega_N \\ \lambda_G & \text{if there is diffusion and } l \in \Omega_S \end{cases} \tag{18}$$

From (13) we get $p_{ml}/w_l = (BC_T)^{1/\beta} \eta_l^{-\theta/\beta}$. (Note that if there is no diffusion, then this expression is not well defined for the South since in that case $\eta_l = 0$.) From (11) we finally get

the real wage for country l, namely

$$w_l/p_l = (AC_{NT})^{-1} (BC_T)^{-(1-\alpha)/\beta} \eta_l^{\theta(1-\alpha)/\beta+\gamma} \qquad (19)$$

3.2.2 Frictionless trade

Turning to the characterization of the equilibrium under frictionless trade, it is convenient to introduce the notions of "national i goods," "north goods," and "global goods" (this is relevant only for intermediate goods). National i goods are those for which the best idea is a national idea in country i (i.e., $x_i = \arg\min\{x\}$); north goods are those for which the best idea is a north idea (i.e., $x_N = \arg\min\{x\}$); and global goods are those for which the best idea is a global idea (i.e., $x_G = \arg\min\{x\}$).

Thanks to diffusion and the fact that trade is frictionless, the equilibrium may entail wage equalization across all countries. In this case all countries produce global goods and all north countries produce north goods. If diffusion is not too strong, then wage differences arise between North and South, and even among north countries. For example, the equilibrium could exhibit an inferior wage in the South, with wage equalization only among north countries. In this equilibrium all north countries produce north goods, and only the South produces global goods. If North-North diffusion is weak relative to differences in research intensities across the North then wage differences would arise among north countries. In this case, there would be a group of north countries with the highest research intensities specializing in the production of their "national goods," with wages determined by each country's research intensity (as in Eaton and Kortum, 2002, and Alvarez and Lucas, 2005), and then a group of north countries with the lowest research intensities sharing a common wage and producing north goods. The wage in the South could be the same as this "low north wage" or it could be lower still. In the later case, all global goods would be produced in the South.

To simplify the exposition, I will focus on the equilibrium with two wage levels: a low wage in the South and a common wage for north countries. This equilibrium is possible even if the research intensity differs among north countries: thanks to diffusion, north countries with low research intensities can specialize in north goods and attain trade balance in spite of the fact that their stock of national ideas per person is relatively low. The key for this equilibrium configuration is that all north countries produce north goods. Under frictionless trade, this requires that the unit cost of the input bundle be equal across countries, i.e. $c_i = Bw_i^\beta p_{mi}^{1-\beta}$ for all $i \in \Omega_N$, so that there is indifference about where to buy north goods. I refer to this

condition as the ECN condition. Since under frictionless trade we have $p_{ml} = p_m$ for all l, then this condition entails $w_i = w_N$ for all i.

In equilibrium, country i will at least supply the whole world of national i goods. Using (8), (9), and (10), and letting $R_i \equiv \phi_i L_i$ and $R_N \equiv \sum_i R_i$, the share of national i goods among all goods is

$$\lambda_i / (\sum \lambda_i + \lambda_N + \lambda_G) = \frac{R_i / R_N}{1 + \widetilde{\delta} + \delta'/g_L}$$

Given the absence of trade costs and the ECN condition, this is also the share of each country's total spending that will be allocated to buying national i goods from country i. Letting $\phi_N \equiv R_N / L_N$ (with $L_N \equiv \sum_i L_i$) be the average research intensity in the North, then a condition necessary for an equilibrium with wage equalization among north countries is

$$\phi_i / \phi_N \leq 1 + \widetilde{\delta} \tag{20}$$

This inequality ensures that - given the ECN condition - every north country has some resources left over for producing north goods. This requires that ϕ_i / ϕ_N be not too high, for otherwise there would be a country that would have so many national goods that it would not be able to satisfy the world demand for its national goods given the ECN condition, and the equilibrium could not take the form that I have postulated here. Note that the condition is relaxed as $\widetilde{\delta}$ increases. This is because a higher δ implies that a lower share of goods are national goods.

The wage in the South relative to the North can be obtained from the conditions $\frac{L_S w_S}{L_N w_N} = \frac{D_S}{1 - D_S}$ and $D_S = \left(c_S^{-1/\theta} / \hat{\psi} \right) \lambda_G$. This yields

$$w_S / w_N = \left(\frac{\widetilde{\delta} \delta'}{\left(1 + \widetilde{\delta} \right) g_L} \frac{L_N}{L_S} \right)^{1/(1 + \beta/\theta)} \tag{21}$$

For the conjectured equilibrium with $w_S < w_N$ we then need the following condition:

$$\delta \leq g_L L_S / L_N \quad \text{or} \quad \delta > g_L L_S / L_N \ \text{together with} \ \delta' < \frac{L_S \left(g_L + \delta \right) g_L}{\delta L_N - g_L L_S} \tag{22}$$

The first inequality or the second and third inequalities together imply that the stock of global ideas is too low, so $w_S / w_N < 1$.

If conditions (20) and (22) are satisfied, then there is an equilibrium of the form that I have conjectured (i.e., wage equalization in the North and $w_S < w_N$). For future reference, note that as diffusion increases then wages in South and North become equalized (i.e. $w_S = w_N$).

Formally, if $\delta > g_L L_S / L_N$ then there exists a $\bar{\delta}'$ such that wages are equalized if $\delta' \geq \bar{\delta}'$. Similarly, if $\delta' > g_L L_S / L_N$ then there exists a $\bar{\delta}$ such that wages are equalized if $\delta \geq \bar{\delta}$.[18] Similarly, wages become equalized for any non-zero δ and δ' if South is sufficiently small.

Now, from (12) and (13) we get

$$p_m = (BC_T)^{1/\beta} w_N \left(\sum_j \lambda_j + \lambda_N + (w_S/w_N)^{-\beta/\theta} \lambda_G \right)^{-\theta/\beta} \tag{23}$$

and from (11) and (23) we get the real wage in north country i,

$$w_N/p_i = (AC_{NT})^{-1} (BC_T)^{-(1-\alpha)/\beta} \left(\sum_j \lambda_j + \lambda_N + (w_S/w_N)^{-\beta/\theta} \lambda_G \right)^{\theta(1-\alpha)/\beta} (\lambda_i + \lambda_N + \lambda_G)^{\gamma} \tag{24}$$

The corresponding result for the South is

$$w_S/p_S = (AC_{NT})^{-1} (BC_T)^{-(1-\alpha)/\beta} \left((w_N/w_S)^{-\beta/\theta} \left(\sum_j \lambda_j + \lambda_N \right) + \lambda_G \right)^{\theta(1-\alpha)/\beta} \lambda_G^{\gamma} \tag{25}$$

3.2.3 Gains from diffusion and frictionless trade

The overall gains from openness for north country i can be seen as the increase in the real wage from the case with no trade and no diffusion to the case with diffusion under frictionless trade.[19] From (19) with $\eta_i = (1 + \delta/g_L)\lambda_i$ and (24), and using the expressions for λ_i, λ_N and λ_G in equations $(8) - (10)$, the gains from openness for north country i are

$$GO_i = r_i^{-\theta(1-\alpha)/\beta} \left(1 + \frac{(w_N/w_S)^{\beta/\theta} - 1}{(g_L + \delta)/\delta'\widetilde{\delta}} \right)^{\theta(1-\alpha)/\beta} \left(\frac{\delta/r_i + g_L}{\delta + g_L} \right)^{\gamma} \tag{26}$$

where $r_i \equiv R_i/R_N$ is the share of worldwide research done by i. The first term captures the gains associated with North-North trade of intermediate goods and diffusion of T ideas; the second term captures the gains from trading with the South; and the final term captures the gains from diffusion of NT ideas. Clearly, countries that account for a smaller share of worldwide research have more to gain from openness. Moreover, as long as δ' is not too low, then $\delta \to \infty$ implies that $w_N = w_S$ and hence $GO_i \to r_i^{-\theta(1-\alpha)/\beta-\gamma}$. Since $\theta(1-\alpha)/\beta + \gamma = g/g_L$, this coincides with the simple logic pursued in the Introduction to compute the gains from openness

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[18]These values for $\bar{\delta}$ and $\bar{\delta}'$ are defined by 21 for $w_S/w_N = 1$.

[19]It is worth emphasizing that in the model there are no transition dynamics since both the benefits from trade and diffusion take place instantaneously. Thus, the increase in the real wage from isolation to openness is the appropriate measure of the welfare gains from openness.

as a pure scale effect in a quasi-endogenous growth model. Of course, if diffusion is finite, then the gains from openness would be smaller than what this simple calculation suggests.

What is the contribution of trade to these gains from openness? The problem in decomposing the overall gains from openness into the contributions of trade and diffusion (of T ideas) is that these two channels are substitutes, in the sense that if one is present then the other one is less important. To see this, it is best to start with an extreme case in which trade and diffusion of T ideas among north countries are *perfect substitutes*. Consider the case in which $\phi_i = \phi$ for all i (symmetry) and again let $\delta \to \infty$ (with δ' not too low). Assume also that $\gamma = 0$ (no NT ideas) to simplify the exposition. Recall that the stock of ideas originated in country i is R_i/g_L. Thus, if all countries shut down diffusion it is easy to show that the real wage under frictionless trade in country i is given by

$$w_i/p_i = A^{-1} \left(BC_T\right)^{-(1-\alpha)/\beta} \left(R_N/g_L\right)^{\theta(1-\alpha)/\beta} \tag{27}$$

On the other hand, $\delta \to \infty$ implies that for all i we have $\lambda_i + \lambda_N + \lambda_G \to R_N/g_L$, so using (19) we see that under autarky but with diffusion the real wage is also given by (27). Thus, in this extreme case, the contribution of diffusion is zero when there is trade, and the contribution of trade is zero when there is diffusion. The general result under normal conditions (i.e., trade is costly and diffusion is finite) is that trade and diffusion are substitutes in the sense that if there is diffusion (trade) then the gains from trade (diffusion) are lower than if diffusion (trade) is not present. Intuitively, imports allow a country to benefit from foreign ideas that have not yet diffused, and diffusion allows a country to benefit from foreign ideas even without trade; diffusion acts as a substitute for trade in the international exchange of ideas among north countries. Another way to state this is that shutting down trade leads a country to rely more on diffusion and this attenuates the resulting losses.

This result implies that it is to some extent arbitrary to decompose the overall gains from openness into separate contributions of trade and diffusion. But it is still meaningful to ask how a country would lose by shutting down trade. Equivalently, we can ask how a country gains by going from a case with diffusion and autarky to a case with diffusion and frictionless trade. The result can then be compared to the calculated gains from trade in a model without diffusion (as in Section 2) and to the overall gains from openness. This entails comparing (19)

with $\eta_i = \lambda_i + \lambda_N + \lambda_G$ to (24), which yields (again, using (8) − (10))

$$GT_i = \left(\frac{1 + \delta/g_L + \left[(w_N/w_S)^{\beta/\theta} - 1 \right] \delta' \widetilde{\delta}/g_L}{r_i + \delta/g_L} \right)^{\theta(1-\alpha)/\beta} \tag{28}$$

Countries with a lower share of worldwide research gain more from trade. Moreover, the gains from trade for north countries are increasing in w_N/w_S, as this reduces their relative cost of procuring global goods. This implies that an increase in the size of the South enlarges the gains from trade for the North. This is a standard terms of trade effect. The result in (28) also shows that, holding w_N/w_S constant, an increase in the rate of North-South diffusion δ' increases the gains from trade for the North. Of course, as shown in (21), an increase in δ' increases the South's relative wage. Thus, an increase in δ' has two opposite effects on the North's gains from trade: on the one hand, it allows more goods to be produced cheaply in the South, which is beneficial to the North, but on the other hand this generates a terms of trade loss for north countries. If δ is not too low then GT_i behaves like an inverted U with respect to δ': when diffusion is low then higher diffusion benefits the North, and the opposite occurs when diffusion is high.[20] Thus, the gains from trade for north countries may either increase or decrease as North-South diffusion increases. If this relationship is positive, then we can say that North-South diffusion and trade behave complements.

We can now explore further the implications of diffusion for the gains from trade by studying the impact of δ on GT_i. Consider first the case in which wages are equalized between North and South. The result that trade and diffusion are substitutes can be appreciated clearly in this case by noting from (28) (with $w_N/w_S = 1$) that GT_i decreases with δ. Another way to see this is to note from the first term of (26) that the joint gains from trade and diffusion of T ideas among north countries are $r_i^{-\theta(1-\alpha)/\beta}$, which does not depend on δ. Since the real wage in the North increases under autarky with δ, then necessarily GT_i will decrease with δ.

Consider now the case in which there is no wage equalization between North and South. Again, there are two opposite effects, because an increase in δ lowers the gains from North-North trade but – if δ is sufficiently small – it increases the North's gains from trading with

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[20]This just entails differentiating the expression for GT_i w.r.t. δ' and noting that this derivative is positive for $\delta' = 0$ and negative for δ' close to the value at which wages are equalized (which exists as long as $\delta > g_L L_S/L_N$). By continuity, there exists a value of δ' at which the derivative of GT_i w.r.t. δ' is zero. Denote this value of δ' by δ'_0. Simple math shows that the second derivative of GT_i w.r.t. δ' evaluated at δ'_0 is negative, estalishing the inverted U shape of GT_i w.r.t. δ'. If $\delta \leq g_L L_S/L_N$ then wages do not become equalized even for high δ', so GT_i may be always increasing in δ'.

the South. Thus, with no wage equalization between North and South, it is conceivable that higher North-North diffusion leads to higher gains from trade in north countries.

Turning now to the gains from trade for the South (the gains from openness are not well defined because under isolation the stock of ideas in the South is zero, implying zero income). This is given by

$$GT_S = \left(\frac{(w_N/w_S)^{-\beta/\theta} (\delta' + g_L + \delta) + \delta\delta'/g_L}{\delta\delta'/g_L} \right)^{\theta(1-\alpha)/\beta} \tag{29}$$

If $w_S = w_N$ then GT_S is decreasing in both δ and δ', implying that trade and diffusion are substitutes for the South. In the general case, we again have conflicting effects: on the one hand, diffusion implies that the South has less to gain from the North because in a sense it already has many of the North's technologies, but on the other hand higher diffusion implies an improvement in the terms of trade for the South, leading to higher gains from trade with north countries. It is easy to show that GT_S behaves like an inverted U with respect to either δ and δ': when diffusion is low, the terms of trade effect dominates, whereas for high diffusion the substitution effect dominates.

4   Calibration

To explore the quantitative implications of the model, I need to choose values for the parameters α, β, θ, γ, δ and δ'. I also need to choose a reasonable value for L_N/L_S, as this pins down the relative wage w_N/w_S, which is necessary to determine the gains from trade. Again, I follow Alvarez and Lucas (2005) and set $\alpha = 0.75$, and $\beta = 0.5$. To assign a value to θ I use a procedure that is similar to Eaton and Kortum (2002), but amended to account for the effect of diffusion. In particular, I assume that trade among north countries can be seen as an equilibrium outcome of the model presented in the previous section (with trade costs) for the particular case in which there is no trade among north countries in north goods. That is, I think of the trade data as coming from the equilibrium of the model for a set of research intensities and trade costs for which each north country satisfies its own demand for north goods with domestic production. I refer to this as the NTNG condition. Recalling the definition $\widetilde{c}_l^N = \min_i \{c_i/k_{li}\}$, the NTNG condition entails $\widetilde{c}_i^N = c_i$ for all i.

One case in which the NTNG condition is satisfied entails a common research intensity across north countries. To gain some intuition for this result, consider the case of frictionless

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trade. With a common research intensity in the North, the absence of trade costs implies that in equilibrium all north countries have the same wage and the same unit cost for the input bundle, i.e. $c_i = c_j$ for all $i, j \in \Omega_N$. In turn, this implies that in equilibrium one can have every north country satisfy its own demand for north goods. If trade costs are positive, then *a fortiori* the NTNG condition will be satisfied. In Appendix A I prove this result for parameters θ and β that satisfy $1/\theta > (1 - \beta)/\beta$ (a condition satisfied in the calibration below) under common research intensities (i.e., $\phi_i = \phi_j$ for $i, j \in \Omega_N$).

The assumption of a common research intensity across north countries is only necessary for the NTNG condition to be satisfied for *any* structure of trade costs k_{ij}. Alternatively, one could assume a simple structure of trade costs with $k_{ij} = k$ for all $i \neq j$, and then find the upper bound $\bar{k}(\phi_1, \phi_2, \ldots, \phi_I, L_1, L_2 \ldots L_I, L_S)$ such that if $k \leq \bar{k}(\phi_1, \phi_2, \ldots, \phi_I, L_1, L_2 \ldots L_I, L_S)$ then the NTNG condition to is satisfied given a vector of research intensities and country sizes $(\phi_1, \phi_2, \ldots, \phi_I, L_1, L_2 \ldots L_I, L_S)$. Clearly such an upper bound for k exists since the NTNG condition is satisfied for k close to zero (see Appendix A).

I now assume that the combination of research intensities, country sizes and trade costs are such that the NTNG condition is satisfied. Although clearly this is not an appropriate characterization for the whole world, it is a reasonable assumption to characterize trade among the richest countries. I also assume that $\widetilde{c}_l \equiv \min_j \{c_j/k_{lj}\} = c_S/k_{lS}$ for all l, so that the South produces all global goods. Applying the equilibrium characterization derived in the previous section to the case with $\widetilde{c}_i^N = c_i$ for all i (NTNG) and $\widetilde{c}_l = c_S/k_{lS}$ for all l we see that now the price index of intermediate goods in north country i is given by

$$p_{mi} = C_T \widetilde{\psi}_i^{-\theta} \tag{30}$$

where

$$\widetilde{\psi}_i = \sum_j \widetilde{\psi}_{ij} \text{ and } \widetilde{\psi}_{ij} = \begin{cases} (c_j/k_{ij})^{-1/\theta} \lambda_j & if & i \neq j \text{ and } j \in \Omega_N \\ c_i^{-1/\theta}(\lambda_i + \lambda_N) & if & i = j \\ (c_S/k_{iS})^{-1/\theta} \lambda_G & if & j = S \end{cases} \tag{31}$$

Trade shares are given by $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$. The relationship between normalized import shares (D_{ij}/D_{jj}) and trade costs $(p_{mj}/p_{mi}k_{ij})$ can now be shown to be (in logs)

$$m_{ij} = -\ln\left(1 + \widetilde{\delta} R_N/R_j\right) - (1/\theta)\kappa_{ij} \tag{32}$$

This is similar to the (normalized) gravity equation in (7) except that now there are source-country fixed effects. These effects are more important for small countries (i.e., m_{ij} is more

negative when R_N/R_j is higher) because in such countries an important part of domestic production is related to north goods, which are not exported to other north countries. This decreases the imports by any country from small countries in relation to (or normalized by) those countries' domestic purchases.

Using the same data on trade volumes and trade costs as Eaton and Kortum (2002), and 1990 R&D employment as a proxy for aggregate research levels R_j and R_N in (32), I estimated $\widetilde{\delta}$ and θ from this equation using non-linear least squares among 19 OECD countries and $g_L = 0.048$.[21] Both parameters are precisely estimated. The estimate of $1/\theta$ is 4.57 with a s.e. of 0.32 while the estimate of $\widetilde{\delta}$ is 0.13 with a s.e. of 0.03.[22] Note that this implies $\theta = 0.22$, significantly higher than Eaton and Kortum's $\theta = 0.12$.

It is worth pausing here to better understand what determines the value of $\widetilde{\delta}$ in this procedure. Equation (32) implies that smaller countries should have lower normalized exports to any other country. This relationship between size as measured by R_j/R_N and normalized exports is what should be pinning down $\widetilde{\delta}$ in the estimation. To see whether this is indeed the case, I ran a linear regression of m_{ij} on κ_{ij} with source-country dummies and compared the exponential of the (negative of the) estimated coefficients for the dummies (which I denote z_j) with R_N/R_j. As shown in Figure 2, there is clear positive relationship between these two variables.[23] This suggests that smaller countries do have lower normalized exports, and that the estimated $\widetilde{\delta}$ in the NLS procedure above is capturing this relationship.

The higher value of θ helps the model better match the observed growth rate even with no technological progress for consumption goods: imposing $\gamma = 0$ in (17) and using $\alpha = 0.75$, $\beta = 0.5$ and $g_L = 4.8\%$, the implied growth rate would now be $g = 0.53\%$ rather than $g = 0.29\%$ obtained in the model without diffusion. But this is still significantly below the observed $g = 1.5\%$. I now set γ to match this growth rate in (17). This yields $\gamma = 0.2$. It is reassuring to note that this value for γ is very close to the value for θ estimated above from an entirely different procedure.

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[21]The R&D employment data comes from the World Development Indicators (WDI) database arranged by the World Bank.

[22]These are robust standard errors. There are 342 observations, and the R-squared is 0.2. The same regression but with no intercept - as in the case of no diffusion - yields an R-squared of 0.06, whereas a simple OLS regression with an intercept yields an R-squared of 0.16. Similar results are obtained if instead I use total population or GDP as proxies for R_i.

[23]A linear regression of z_j on R_N/R_j yields an estimated slope coefficient of 0.26 with a s.e. of 0.09. The outlier is Australia: this country's high z means that it has low normalized exports – a characteristic of small countries (high R_N/R_j) – in spite of being a large country as measured by a low R_N/R_j.

The last step is to calibrate δ and δ'. Eaton and Kortum (1999) calculate diffusion lags from international patent data and find an average mean diffusion lag of ten years among the five leading economies. This may significantly understate the diffusion lag for *all ideas* conducive to trade, however, because only a small subset of technologies are patented and it is reasonable to expect that those technologies are precisely the ones that are likely to diffuse rapidly.[24] Comin, Hobijn and Rovito (2006) estimate diffusion lags for several technologies. They find median diffusion lags that range from 8 years for the Internet to 74 years for cars. Since the average diffusion lag for ideas among rich countries is $1/\delta$, then it seems reasonable to consider $\delta \in [0.01, 0.1]$, with the corresponding value of δ' adjusted to keep $\widetilde{\delta} = \delta/(\delta' + g_L) = 0.13$.[25]

To apply the formula for the gains from openness in (26) I need a value for the wage in the North relative to the South under frictionless trade. For each value of $\delta \in [0.01, 0.1]$ I adjust the relative size of the North (L_N/L_S) so that under frictionless trade it accounts for 75% of world GDP (i.e. $\frac{w_N L_N}{w_N L_N + w_S L_S} = 3/4$) given the relative wage in (21).[26] The assumption here is that the North's share of world GDP would not be too different under frictionless trade compared to the case with actual trade barriers. Figure 3 depicts w_N/w_S for $\delta \in [0.01, 0.1]$. The highest value of w_N/w_S is attained for the case with the lowest rate of diffusion: $\delta = 0.01$ implies $w_N/w_S = 2$. This reveals that the model is not able to generate large TFP differences across countries; such differences would have to come from the international variation in the effective quantity of resources (L in the model) per worker.

An important implication is that wages become equalized across North and South for $\delta \geq 0.024$. In this case, the North would produce some global goods, which is not consistent with the equilibrium that I considered for the calibration of θ and $\widetilde{\delta}$, where I assumed that all global goods were produced in the South. Thus, for the analysis below I consider only the restricted range $\delta \in [0.01, 0.024]$. For such values of δ wages satisfy $w_S < w_N$ and the South produces all global goods.

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[24] See for instance Cohen, Nelson and Walsh (2000).

[25] The reader may worry about the implication of low rates of diffusion for the existence of national scale effects. That is, one could expect that if δ is low then small countries would be less productive than large ones because of a smaller stock of ideas. Although trade would greatly reduce such scale effects for T ideas, this would not help for NT ideas. But this is only a problem if one assumes that consumption goods are tradable across all points within a country. A more reasonable assumption is that such goods are non-tradable across national subregions. Under these conditions scale effects would not arise at the national level.

[26] This is the share of worldwide GDP accounted for by the 19 OECD countries used by Eaton and Kortum (2002) in their estimation of θ. The GDP data comes from the WDI, World Bank, average 1994-2000 (taken from Alvarez and Lucas, 2005).
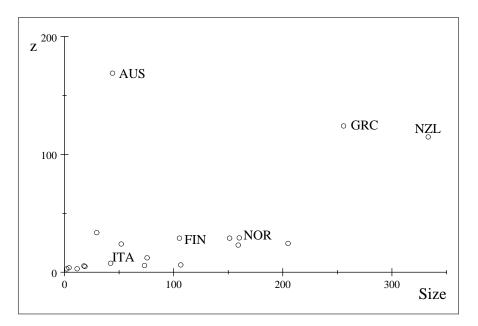
Figure 2: Source country dummies (z, vertical axis) versus size (R_N/R_j, horizontal axis)

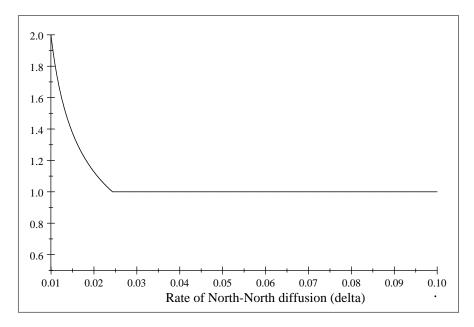Figure 3: Relative wage for the North w_N/w_S for different values of δ

5   Gains from trade and diffusion: quantitative results

I now use the parameters calibrated in the previous section to compute values for the gains from openness and the gains from frictionless trade for a country that does 1% of the world's total research. I then generalize to countries with different shares of world research.

Applying the formula in (26) yields gains from openness for a country with $r_i = 1\%$ that range between 206% to 240% (i.e., $GO_i \in [3.06, 3.4]$) as δ increases from 0.01 to 0.024. Figure 4 shows these gains (GO) as well as the first and third terms of GO_i in (26) – the second term is not shown because it too small relative to the other terms in the figure. The first term, $GO1$, measures the gains from trade and diffusion of T ideas between north countries, whereas the third term, $GO3$, measures these countries' gains from North-North diffusion of NT ideas. The overall gains from openness are large relative to the gains from openness in the model without diffusion (see Section 2). Partly, this is a result of the gains from diffusion of NT ideas ($GO3$), which ranges from 80% to 104% as δ increases from 0.01 to 0.024. But this also comes from the increase in the estimated value of θ when diffusion is allowed into the model, as can be verified by noting that $GO1$ increases from 1.32 to 1.66 as θ increases from 0.12 to 0.22.
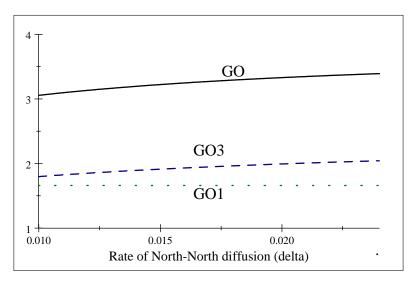

Figure 4: Gains from Openness (GO), Gains from North-North Trade and Diffusion of T ideas (GO1), and Gains from North-North Diffusion of NT ideas (GO3) for different values of δ.

What is the role of trade in generating these gains? Applying (28) we find that the gains from trade for a country with $r_i = 1\%$ range from 23.7% to 13% as δ increases from 0.01 to 0.024, as illustrated in Figure 5. These gains seem small compared to the large overall gains

from openness calculated above. They are also smaller than the gains from trade that would arise in a model without diffusion. In this case, and with symmetric research intensities (i.e., $\phi_i = \phi$ for all i), then $GT_i = 1.66$. An important implication is that although diffusion brings about gains from trade with the South, these extra gains are dominated by the substitutative property between North-North trade and diffusion; in other words, although in theory trade and diffusion could be complements via trade with the South, this is not the case for the set of parameters considered here.[27]
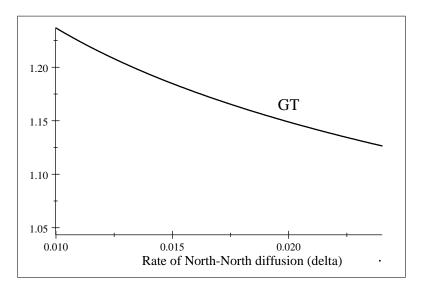

Figure 5: Gains from Trade (GT) under different values of δ

The previous calculations have been made for the case of $r_i = 1\%$. Figure 6 shows similar results for a range of different levels of r for $\delta = 0.01$.[28] This relatively low rate of diffusion is chosen to be on the conservative side regarding the gains from openness and to allow for higher gains from trade. Figure 7 shows the relationship between the gains from trade in this model and the gains from trade in the Alvarez and Lucas model (i.e., $\theta = 0.15$ and no diffusion). The result presented above for $r_i = 1\%$ that the gains from trade in the model with diffusion are lower than the AL model remains valid as long as r is not too high; for very large countries

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[27]To see this in more detail, consider an intermediate situation with diffusion but no wage differences between North and South. Imposing $w_N = w_S$ in (28) we get $GT_i \in [1.126, 1.207]$. This reveals that the gains from trade drop from 66% to between 12.6% and 20.7% as we introduce diffusion but retain wage equalization; allowing for wage differences between North and South increases the gains from trade only slightly. Thus, diffusion decreases the gains from trade in spite of the fact that it leads to "new" gains from trading with the South.

[28]Again, the second term of GO_i in (26) – which corresponds to the gains from North-South trade – is not plotted because it is too small relative to $GO1$ and $GO3$ ($GO2 = 1.025$ for any r_i).

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(i.e., $r \geq 10\%$), the larger θ in the model with diffusion dominates, leading to higher gains from trade than in the AL model.
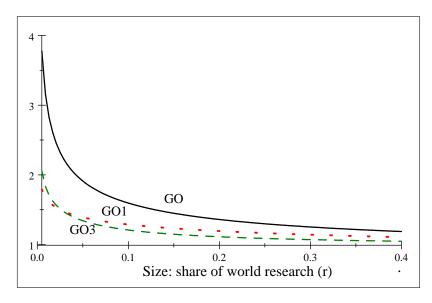

Figure 6: Gains from Openness (GO), Gains from North-North Trade and Diffusion of T ideas (GO1), and Gains from North-North Diffusion of NT ideas (GO3) against size (r) for $\delta = 0.01$.

Finally, I turn to the gains from trade for the South (the gains from openness are not well defined because its income is zero under isolation). Applying (29) reveals that GT_S goes from 1.23 to 1.55 as δ increases from 0.01 to 0.024. The gains from trade for the South become larger as diffusion raises: this implies that the improvement in the South's terms of trade dominates the substitution effect as δ and δ' increase and L_N/L_S is recalibrated to keep the North's share of world GDP.

6   Gains from trade and openness under transportation costs

How are the gains from trade and openness affected by transportation costs? How do these gains differ across OECD countries? To answer these questions, in this section I undertake an alternative quantitative exercise. First, I assume that $\theta = \gamma$ and calibrate this common parameter to match the observed growth rate. From (17) this yields $\theta = \gamma = 0.21$, which is close to the values $\theta = 0.22$ and $\gamma = 0.2$ estimated in Section 4 and used to derive the results of Section 5. Second, I assume that $\lambda_i = L_i$, with L_i measured by the number of researchers
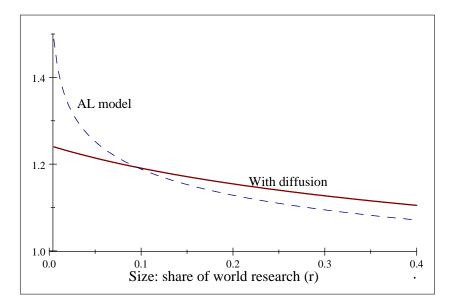
Figure 7: Gains from Trade in the Model with Diffusion and the AL Model against size (r) for $\delta = 0.01$.

in country i. And third, instead of matching the way in which trade costs affect trade volumes (gravity equation), I assume that $k_{ij} = k = 0.75$ for $i \neq j$, and choose values for $\widetilde{\delta}$ and L_S so that the simulated equilibrium (see below) matches the share of the South in world GDP (25%) and the average import share of the 19 OECD countries considered above (30%), while having the South produce all goods with global technologies (as in the calibration of Section 4).[29],[30] In particular, I set $\delta = 0.01$, $\widetilde{\delta} = 0.18$ and $L_S = \sum_i L_i$. Note that $\widetilde{\delta} = 0.18$ is not too different from the estimated value in Section 4 (i.e., $\widetilde{\delta} = 0.13$).

To compute the gains from trade and openness in the presence of trade barriers, I need to compute equilibrium real wages in an equilibrium with trade barriers and then compare them to the real wages in isolation and autarky derived above. The algorithm I use to compute the equilibrium is an extension of the one developed by Alvarez and Lucas (2005) for an economy without diffusion. As explained in Appendix A, one element of the equilibrium concerns the way in which South allocates its expenditures of goods produced with North technologies ("North goods") across North countries. This allocation is captured by the vector $x = (x_1, x_2, ...x_I)$, where x_l is the share of South expenditures on North goods bought from country l. The vector x

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[29] Alvarez and Lucas (2005) use $k = 0.75$ as a compromise between the low values estimated in gravity equations and direct estimates of transportation costs (freight charges plus the time costs of cargo in transit).

[30] The data on total imports and GDP is from the WDI, World Bank, for the average of 1994 - 2000 (Alvarez and Lucas, 2005).

belongs to the I-dimensional simplex (i.e., $x \in \Delta^I$) and in equilibrium must satisfy the following "complementary slackness" conditions: 1) if $x_i, x_j > 0$ then $c_i/k_{Si} = c_l/k_{Sj}$, 2) if $x_i = 0$ and $x_j > 0$ then $c_i/k_{Si} \geq c_j/k_{Sj}$. Appendix B explains the algorithm to compute x together with the rest of the equilibrium.

The results are illustrated in Figures 8 and 9, and Table 1 contains the full simulation results. South buys North goods from all North countries except the two largest countries, U.S. and Japan, which have the highest wages and unit costs. Figure 8 shows that the model's implied imports to GDP ratios line well against the data (the outlier is Belgium). Figure 9 shows how gains from openness vary with country size; New Zealand exhibits gains above 290%. More importantly, the average gains from trade among North countries are 9%, whereas the average gains from openness are 143% (Table 1). Not surprisingly, the gains from trade and openness are smaller than those calculated under frictionless trade, and the result that GO are significantly higher than GT remains valid here when we allow for transportation costs.
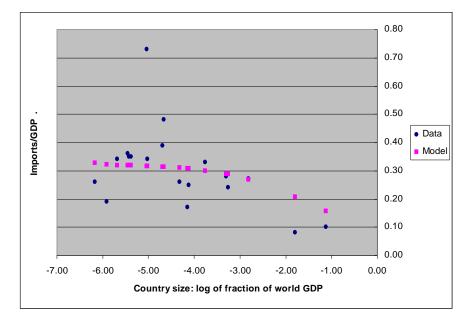

Figure 8: Imports/GDP in the model and data against country size

7   Conclusion

Countries benefit from openness to the rest of the world in many different ways. Trade and diffusion are surely two of the most important channels for the realization of these benefits.
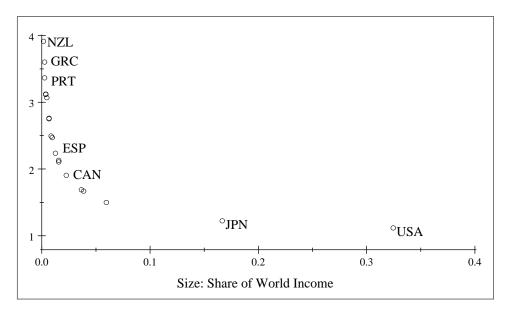
Figure 9: Gains from Openness vs Size

Table 1: Results for simulations for k = 0.75, $\delta = 0.01$, $\widetilde{\delta} = 0.18$, $L_s = \sum L_i$. (1) is for model results and (2) for data.

Country	Wage	Unit cost	x	Income (1)	Income (2)	M/GDP (1)	M/GDP (2)	GT	GO
Australia	0.55	1.07	7.6%	1.6%	1.3%	31%	17%	1.09	2.12
Austria	0.55	1.07	2.6%	0.5%	0.7%	32%	35%	1.10	3.06
Belgium	0.55	1.07	3.6%	0.7%	0.9%	32%	73%	1.09	2.75
Canada	0.55	1.07	10.1%	2.3%	2.1%	30%	33%	1.09	1.90
Denmark	0.55	1.07	2.5%	0.4%	0.6%	32%	35%	1.10	3.11
Spain	0.55	1.07	6.6%	1.3%	1.9%	31%	26%	1.09	2.23
Finland	0.55	1.07	3.6%	0.7%	0.4%	32%	34%	1.09	2.74
France	0.55	1.07	12.9%	3.9%	4.9%	29%	24%	1.08	1.66
U.K.	0.55	1.07	12.7%	3.7%	4.3%	29%	28%	1.08	1.68
Germany	0.56	1.07	12.7%	6.0%	7.4%	27%	27%	1.07	1.49
Greece	0.55	1.07	1.6%	0.3%	0.4%	32%	19%	1.10	3.59
Italy	0.55	1.07	7.8%	1.6%	3.9%	31%	25%	1.09	2.10
Japan	0.59	1.08	0.0%	16.7%	15.7%	21%	8%	1.05	1.22
Netherlands	0.55	1.07	5.0%	1.0%	1.3%	31%	48%	1.09	2.46
Norway	0.55	1.07	2.4%	0.4%	0.5%	32%	36%	1.10	3.11
New Zealand	0.55	1.07	1.6%	0.2%	0.2%	32%	26%	1.10	3.91
Portugal	0.55	1.07	1.9%	0.3%	0.4%	32%	34%	1.10	3.36
Sweden	0.55	1.07	4.8%	0.9%	0.8%	31%	39%	1.09	2.49
USA	0.62	1.09	0.0%	32.5%	28.0%	16%	10%	1.03	1.11
North	0.55	1.07	...	...	75.0%	30%	30%	1.09	2.43
South	0.20	0.62	...	25.0%	25.0%	28%	...	1.06	...

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Building on Eaton and Kortum (2001, 2002) and Alvarez and Lucas (2005) I developed a model of trade and diffusion where growth is caused by technological progress in tradable and non-tradable goods. Calibrating the model to match the trade data and the observed growth rate I then computed the model's implied gains from openness and the gains from trade. The main result is that the gains from openness are quite large, whereas the gains from trade are even smaller than in the model without diffusion.

One concern is that the analysis implicitly assumes that the flow of ideas is independent of the volume of trade. Perhaps trade has a much larger role precisely through a positive effect on diffusion. This could happen through several channels, such as flows of ideas arising from the interaction between nationals and foreigners via trade, the competitive pressure from imports inducing domestic firms to engage in faster technology adoption, or complementarities between foreign technologies and foreign inputs.[31] There is a large empirical literature exploring the significance of these and similar mechanisms through which trade may induce productivity growth in domestic firms.[32] This is an important topic for future research.

A limitation of the model concerns the way in which diffusion is captured. There are at least three specific tasks ahead. First, assuming simultaneous diffusion, so that national ideas become available in all rich countries and then globally, is clearly unrealistic. It seems important to model diffusion according to the qualitative and hopefully quantitative features found in the data (see for example Jaffe and Trajtenberg, 1999, Keller, 2002, and Comin et. al. 2006) and explore the implications of this for growth and trade.

Second, FDI is surely one mechanism through which diffusion takes place. Ramondo (2006) has already shown a way to model FDI within the Eaton and Kortum framework when there is no trade. The next step is to have a model with trade, FDI and (pure) diffusion and quantify the gains from each of these channels separately. Including migration into such a model would be another worthwhile step.

Finally, this paper has ignored the importance of differences across countries in technology adoption. One conjecture that would be interesting to explore is that countries with lower rates of technology adoption have less to gain from diffusion and more to gain from trade, with lower

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[31] We tend to see countries that are closed to trade also being closed to foreign ideas (e.g., North Korea). But this empirical association between trade and diffusion does not imply any causal role for trade in accelerating diffusion.

[32] See for instance Rhee et. al. (1984), Aitken et. al. (1997), Clerides et. al. (1998), Bernard and Jensen (1999), Hallward-Driemeier et. al. (2002), Tybout (2003), Atkeson and Burstein (2007).

overall gains from openness.

A final remark concerns the assumption that research efforts are exogenous. How would the results change if this assumption were relaxed? In the simplest model with no diffusion of T ideas, as in Eaton and Kortum (2001), trade does not affect countries' research intensity (i.e., the share of the labor force devoted to research). The reason for this is the standard one that although trade expands the market for ideas, it also increases competition, and these two effects exactly balance out. This implies that, to a first approximation, the gains from trade would not be affected by having endogenous research efforts. Something similar happens with the gains from diffusion. To see this, consider the extreme case in which diffusion is instantaneous (no frictions to the international diffusion of ideas, i.e. $\delta, \delta' \to \infty$) and research productivities are equal across north countries (i.e., $\phi_i = \phi$ all i). Shutting down diffusion would imply larger returns to ideas in the home market, but would prevent the exploitation of ideas in foreign markets. As shown in Eaton and Kortum (2006), these two effects exactly cancel out, so diffusion has no effect on innovation. This discussion suggests that the results obtained here with exogenous research efforts would not be significantly affected by the extension to endogenous research. Still, a thorough analysis of this issue seems worthwhile, and is left for future research.

Appendix A

This Appendix shows that equations (30) and (3) with $D_{ij} = \widetilde{\psi}_{ij}/\widetilde{\psi}_i$ and (31) are part of the system of equations that characterize the full equilibrium conditions for the case with $\phi_i = \phi_j$ for all $i, j \in \Omega_N$ and $1/\theta > (1-\beta)/\beta$. I restrict attention to a case in which South supplies all countries with global goods.

The analysis of Section 3.1 shows that for $l \neq i$ and $i, l \in \Omega_N$ import shares are given by

$$D_{li} = \widetilde{\psi}_{li}/\widetilde{\psi}_l = (c_i/k_{li})^{-1/\theta} \lambda_i/\widetilde{\psi}_l = (c_i/k_{li})^{-1/\theta} \lambda_i (p_{ml}/C_T)^{1/\theta} = \mu c_i^{-b} p_{ml}^b a_{li} \lambda_i$$

where $b \equiv 1/\theta$, $\mu \equiv C_T^{-b}$, and $a_{li} \equiv k_{li}^b$, whereas for $l \in \Omega_N$ we have $D_{ll} = \mu c_l^{-b} p_{ml}^b (\lambda_l + \lambda_N)$. Also, imports from the South are related to $D_{lS} = \mu c_S^{-b} p_{ml}^b a_{lS} \lambda_G$. On the other hand, for $l \notin \arg\min \{c_j/k_{Sj}\}$ Southern imports are determined by $D_{Sl} = \mu c_l^{-b} p_{mS}^b a_{Sl} \lambda_l$ whereas for $l \in \arg\min \{c_j/k_{Sj}\}$ we have $D_{Sl} = \mu c_l^{-b} p_{mS}^b a_{Sl} (\lambda_l + x_l \lambda_N)$, where x_l is the share of goods produced with north ideas that country l supplies to South. The vector $x = (x_1, x_2, ...x_I)$ is in the I-dimensional simplex (i.e., $x \in \Delta^I$) and satisfies the following "complementary slackness" conditions: 1) if $x_j, x_l > 0$ then $c_j/k_{Sj} = c_l/k_{Sl}$, 2) if $x_j = 0$ and $x_l > 0$ then $c_j/k_{Sj} \geq c_l/k_{Sl}$. Finally, $D_{SS} = \mu c_S^{-b} p_{mS}^b \lambda_G$.

Using these results, the trade balance condition (i.e., $\sum_j L_j w_j D_{ji} = L_i w_i$) for country $i \in \Omega_N$ is

$$\sum_l L_l w_l \mu c_i^{-b} p_{ml}^b a_{li} \lambda_i + L_i w_i \mu c_i^{-b} p_{mi}^b \lambda_N + L_S w_S \mu c_i^{-b} p_{mS}^b a_{si} (\lambda_i + x_l \lambda_N) = L_i w_i \quad (33)$$

For the South the trade balance condition is

$$\sum_l L_l w_l \mu c_S^{-b} p_{ml}^b a_{lS} \lambda_G + L_S w_S \mu c_S^{-b} p_{mS}^b \lambda_G = L_S w_S \quad (34)$$

On the other hand, the price equations are for the North,

$$p_{mi}^{-b}/\mu = \sum_l a_{il} c_l^{-b} \lambda_l + c_i^{-b} \lambda_N + a_{iS} c_S^{-b} \lambda_G \quad (35)$$

and for the South, letting $\widetilde{c}_{SN} = \min_{j \in \Omega_N} \{c_j/k_{Sj}\}$,

$$p_{mS}^{-b}/\mu = \sum_l a_{Sl} c_l^{-b} \lambda_l + \widetilde{c}_{SN}^{-b} \lambda_N + c_S^{-b} \lambda_G \quad (36)$$

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Together with $c_i = Bw_i^\beta p_{mi}^{1-\beta}$, equations $(33-36)$ characterize the equilibrium as long as

$$c_i \leq c_j/k_{ij} \text{ for all } i, j \in \Omega_N \tag{37}$$

I will refer to this as the NTC condition. In the rest of this Appendix A I show that a solution of $(33-36)$ with $c_i = Bw_i^\beta p_{mi}^{1-\beta}$ necessarily satisfies the NTC condition. A first step is to prove the following lemma:

Lemma 1 $c_i^{-b} \geq a_{ij}c_j^{-b}$ implies $p_{mi}^{-b} \geq a_{ij}p_{mj}^{-b}$.

Proof. First note from (35) that

$$\frac{a_{ij}p_{mj}^{-b}}{p_{mi}^{-b}} = \frac{\sum_l a_{ij}a_{jl}c_l^{-b}\lambda_l + a_{ij}c_j^{-b}\lambda_N + a_{ij}a_{js}c_S^{-b}\lambda_G}{\sum_l a_{il}c_l^{-b}\lambda_l + c_i^{-b}\lambda_N + a_{is}c_S^{-b}\lambda_G}$$

The triangular inequality implies $a_{ij}a_{jl} \leq a_{il}$, which then implies that

$$\frac{a_{ij}p_{mj}^{-b}}{p_{mi}^{-b}} \leq \frac{\sum_l a_{il}c_l^{-b}\lambda_l + a_{ij}c_j^{-b}\lambda_N + a_{is}c_S^{-b}\lambda_G}{\sum_l a_{il}c_l^{-b}\lambda_l + c_i^{-b}\lambda_N + a_{is}c_S^{-b}\lambda_G}$$

Given the assumption $a_{ij}c_j^{-b} \leq c_i^{-b}$ then clearly $a_{ij}p_{mj}^{-b} \leq p_{mi}^{-b}$. ∎

The second step is to prove the following lemma:

Lemma 2 If $x_i = 0$ and $x_j \geq 0$ then $p_{mi}^b a_{ij} \leq p_{mj}^b$.

Proof. To simplify notation, I set $\phi_N = 1$. Then $\lambda_i = L_i$ for all $i \in \Omega_N$. Equation (33) implies

$$\sum_l \lambda_l w_l \mu c_j^{-b} p_{ml}^b a_{jl} + w_j \mu c_j^{-b} p_{mj}^b \lambda_N + L_S w_S \mu c_j^{-b} a_{Sj} p_{mS}^b \left(1 + x_j \frac{\lambda_N}{\lambda_j}\right) = w_j$$

Using (35) to substitute for μp_{mj}^b in the second term we get

$$c_j^b w_j = \mu \sum_l \lambda_l w_l p_{ml}^b a_{jl} + \frac{\lambda_N w_j}{\sum_l a_{jl}c_l^{-b}\lambda_l + c_j^{-b}\lambda_N + a_{jS}c_S^{-b}\lambda_G} + L_S w_S \mu a_{jS} p_{mS}^b \left(1 + x_j \frac{\lambda_N}{\lambda_j}\right)$$

Moving the second term of the RHS to the left and simplifying yields

$$\frac{c_j^b w_j a_{jS}c_S^{-b}\lambda_G + c_j^b w_j \sum_l a_{jl}c_l^{-b}\lambda_l}{\sum_l a_{jl}c_l^{-b}\lambda_l + c_j^{-b}\lambda_N + a_{jS}c_S^{-b}\lambda_G} = \mu \sum_l \lambda_l w_l p_{ml}^b a_{jl} + L_S w_S \mu a_{jS} p_{mS}^b \left(1 + x_j \frac{\lambda_N}{\lambda_j}\right) \tag{38}$$

Using (35) again shows that

$$p_{mj}^b c_j^b w_j \left(a_{jS}c_S^{-b}\lambda_G + \sum_l a_{jl}c_l^{-b}\lambda_l\right) = \sum_l \lambda_l w_l p_{ml}^b a_{jl} + L_S w_S a_{jS} p_{mS}^b \left(1 + x_j \frac{\lambda_N}{\lambda_j}\right)$$

A similar procedure implies

$$p_{mi}^b c_i^b w_i (a_{iS} c_S^{-b} \lambda_G + \sum_l a_{il} c_l^{-b} \lambda_l) = \sum_l \lambda_l w_l p_{ml}^b a_{il} + L_S w_S a_{iS} p_{mS}^b$$

Multiplying by a_{ij} at both sides of this equation and applying the triangular inequality reveals that

$$
\begin{aligned}
a_{ij} p_{mi}^b c_i^b w_i &\leq \frac{\sum_l \lambda_l w_l p_{ml}^b a_{jl} + L_S w_S a_{jS} p_{mS}^b}{a_{iS} c_S^{-b} \lambda_G + \sum_l a_{il} c_l^{-b} \lambda_l} \\
&\leq \frac{\sum_l \lambda_l w_l p_{ml}^b a_{jl} + L_S w_S a_{jS} p_{mS}^b (1 + x_j \frac{\lambda_N}{\lambda_j})}{a_{iS} c_S^{-b} \lambda_G + \sum_l a_{il} c_l^{-b} \lambda_l}
\end{aligned}
$$

Using (38) yields

$$a_{ij} p_{mi}^b c_i^b w_i \leq p_{mj}^b c_j^b w_j \frac{a_{jS} c_S^{-b} \lambda_G + \sum_l a_{jl} c_l^{-b} \lambda_l}{a_{iS} c_S^{-b} \lambda_G + \sum_l a_{il} c_l^{-b} \lambda_l}$$

Multiplying again by a_{ij} and applying the triangular inequality implies

$$a_{ij}^2 p_{mi}^b c_i^b w_i \leq p_{mj}^b c_j^b w_j$$

Using $c_i = B w_i^\beta p_{mi}^{1-\beta}$ then

$$a_{ij}^2 p_{mi}^b \left(\frac{c_i}{p_{mi}^{1-\beta}} \right)^{1/\beta} c_i^b \leq p_{mj}^b \left(\frac{c_j}{p_{mj}^{1-\beta}} \right)^{1/\beta} c_j^b$$

This implies that

$$a_{ij}^2 \frac{p_{mi}^{b-(1-\beta)/\beta} c_i^{b+1/\beta}}{p_{mj}^{b-(1-\beta)/\beta} c_j^{b+1/\beta}} \leq 1$$

Now, suppose by contradiction that $p_{mj}^b < a_{ij} p_{mi}^b$. Lemma 1 then implies $c_j^b < c_i^b a_{ij}$, so given that $b - (1-\beta)/\beta > 0$ then

$$\frac{p_{mi}^{b-(1-\beta)/\beta} c_i^{b+1/\beta}}{p_{mj}^{b-(1-\beta)/\beta} c_j^{b+1/\beta}} > (\frac{1}{a_{ij}})^{2+1/b}$$

This is a contradiction with the above equation and hence establishes that $p_{mi}^b a_{ij} \leq p_{mj}^b$. ∎

Lemma 3 *If $x_i = 0$ and $x_j \leq 0$ then $c_i^b a_{ij} \leq c_j^b$.*

Proof. From (33) and the triangular inequality we get

$$
\begin{aligned}
\frac{w_i c_i^b a_{ij}}{w_j c_j^b} &= \frac{\sum_l \lambda_l w_l \mu p_{ml}^b a_{il} a_{ij} + w_i a_{ij} \mu p_{mi}^b \lambda_N + L_S w_S \mu p_{mS}^b a_{Si} a_{ij}}{\sum_l \lambda_l w_l \mu p_{ml}^b a_{jl} + w_j \mu p_{mj}^b \lambda_N + L_S w_S \mu p_{mS}^b a_{Sj}(1 + x_j \frac{\lambda_N}{\lambda_j})} \\
&\leq \frac{\sum_l \lambda_l w_l \mu p_{ml}^b a_{jl} + w_i a_{ij} \mu p_{mi}^b \lambda_N + L_S w_S \mu p_{mS}^b a_{Sj}}{\sum_l \lambda_l w_l \mu p_{ml}^b a_{jl} + w_j \mu p_{mj}^b \lambda_N + L_S w_S \mu p_{mS}^b a_{Sj}} \\
&\leq \max \left\{ 1, \frac{w_i p_{mi}^b a_{ij}}{w_j p_{mj}^b} \right\}
\end{aligned}
$$

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There are two cases to consider: (1) $\frac{w_i c_i^b a_{ij}}{w_j c_j^b} \leq \frac{w_i p_{mi}^b a_{ij}}{w_j p_{mj}^b}$ and (2) $\frac{w_i c_i^b a_{ij}}{w_j c_j^b} \leq 1$. In case (1) we have

$$\frac{c_i^b a_{ij}}{c_j^b} \leq \frac{p_{mi}^b a_{ij}}{p_{mj}^b}$$

Given lemma 2 then this implies $\frac{c_i^b a_{ij}}{c_j^b} \leq 1$. In case (2) then we have $\frac{w_i}{w_j} \leq \frac{c_j^b}{c_i^b a_{ij}}$. But we know that $c_i = B p_{mi}^{1-\beta} w_i^\beta$, so

$$\frac{c_i^b a_{ij}}{c_j^b} = a_{ij} \left(\frac{p_{mi}^{1-\beta} w_i^\beta}{p_{mj}^{1-\beta} w_j^\beta} \right)^b \leq a_{ij} (\frac{p_{mi}^b}{p_{mj}^b})^{1-\beta} (\frac{c_j^b}{c_i^b a_{ij}})^{b\beta}$$

Hence

$$(\frac{c_i^b a_{ij}}{c_j^b})^{1+b\beta} \leq a_{ij} (\frac{p_{mi}^b}{p_{mj}^b})^{1-\beta}$$

But since $p_{mi}^b / p_{mj}^b \leq 1/a_{ij}$, then

$$\left(\frac{c_i^b a_{ij}}{c_j^b} \right)^{1+b\beta} \leq a_{ij}^\beta$$

This implies that $c_i^b a_{ij} / c_j^b \leq a_{ij}^{\frac{\beta}{1+b\beta}}$, so $c_i^b a_{ij} \leq c_j^b$. ∎

Lemma 3 implies that if $x_i = x_j = 0$ then $c_i^b a_{ij} \leq c_j^b$ and $c_j^b a_{ij} \leq c_i^b$. If $x_i = 0$ and $x_j > 0$ then this lemma implies that $c_i^b a_{ij} \leq c_j^b$. The property $c_j^b a_{ij} \leq c_i^b$ follows easily from the triangular inequality: we have $\frac{c_i}{k_{Si}} \geq \frac{c_j}{k_{Sj}}$, so $\frac{c_i}{k_{ij}} \geq \frac{c_j k_{Si}}{k_{Sj} k_{ij}} \geq \frac{c_j k_{Si}}{k_{Si}} = c_j$, hence $c_i^b \geq c_j^b a_{ij}$. Finally, if $x_i, x_j > 0$ then $c_i / k_{Si} = c_j / k_{Sj}$. Then it is clear from the triangular inequality and symmetry that

$$\frac{c_i}{k_{ji}} = \frac{c_j k_{Si}}{k_{ji} k_{Sj}} = \frac{c_j k_{Si}}{k_{ij} k_{jS}} \geq \frac{c_j k_{Si}}{k_{iS}} = \frac{c_j k_{iS}}{k_{iS}} = c_j$$

Thus $c_j^b a_{ij} \leq c_i^b$, and similarly we can show $c_i^b a_{ij} \leq c_i^b$. Thus, we have shown that for any $x_i, x_j \geq 0$ we have both $c_j \leq c_i / k_{ji}$ and $c_i \leq c_j / k_{ij}$, which shows that the NTC condition is satisfied.

This completes the proof of the Proposition. In the rest of this Appendix I consider the case in which $k_{ij} = k$ ($a_{ij} = a$) for all i, j but $\phi_i \neq \phi_j$ and derive an upper bound for k such that NTC holds. The equilibrium must satisfy

$$p_{mi}^{-b} / \mu = \sum_{l \neq i}^{I} a c_l^{-b} \lambda_l + c_i^{-b} (\lambda_i + \lambda_N) + a c_S^{-b} \lambda_G \tag{39}$$

$$w_l L_l = \sum_{i \neq l}^{I} \lambda_l L_i w_i \mu c_l^{-b} p_{mi}^b a + L_l w_l \mu c_l^{-b} p_{ml}^b (\lambda_l + \lambda_N) + a L_S w_S \mu c_l^{-b} p_{mS}^b (\lambda_l + x_l \lambda_N) \tag{40}$$

Plugging the first into the second equation and simplifying yields

$$p_{ml}^b c_l^b w_l L_l \left(\sum_{i=1}^I c_i^{-b} \lambda_i + c_S^{-b} \lambda_G \right) = \lambda_l \sum_{i=1}^I L_i w_i p_{mi}^b + L_S w_S p_{mS}^b (\lambda_l + x_l \lambda_N)$$

Doing the same for index j shows that

$$\frac{p_{ml}^b c_l^b w_l L_l}{p_{mj}^b c_j^b w_j L_j} = \frac{\lambda_l (\sum_{i=1}^I L_i w_i p_{mi}^b + L_S w_S p_{mS}^b) + L_S w_S p_{mS}^b \lambda_N x_l}{\lambda_j (\sum_{i=1}^I L_i w_i p_{mi}^b + L_S w_S p_{mS}^b) + L_S w_S p_{mS}^b \lambda_N x_j}$$

Given that $x_l, x_j \le 1$ then this implies the following inequality

$$\frac{p_{ml}^b c_l^b w_l L_l}{p_{mj}^b c_j^b w_j L_j} \le \frac{\lambda_l + \lambda_N}{\lambda_j}$$

and hence

$$\frac{w_l}{w_j} \le A_{lj} \left(\frac{p_{ml} c_l}{p_{mj} c_j} \right)^{-b} \tag{41}$$

where $A_{lj} = \frac{(\lambda_l + \lambda_N) L_j}{\lambda_j L_l}$. Using $c_i = B w_i^\beta p_{mi}^{1-\beta}$ then implies

$$\frac{c_l}{c_j} \le A_{lj}^{\frac{\theta \beta}{\theta + \beta}} \left(\frac{p_{mj}}{p_{ml}} \right)^{\frac{\beta - \theta(1-\beta)}{\theta + \beta}}$$

But from (39) we get

$$\frac{p_{ml}^{-b}}{p_{mj}^{-b}} = \frac{\sum_{i=l}^I a c_i^{-b} \lambda_i + c_l^{-b}(\lambda_l(1-a) + \lambda_N) + a c_S^{-b} \lambda_G}{\sum_{i=j}^I a c_i^{-b} \lambda_i + c_j^{-b}(\lambda_k(1-a) + \lambda_N) + a c_S^{-b} \lambda_G}$$

which implies

$$\frac{p_{mj}}{p_{ml}} \le Q \max_{lj} \left(\frac{c_j}{c_l} \right)$$

where $Q \equiv \left[max_{lj}(\frac{\lambda_l}{\lambda_j}) \right]^\theta$. Combining this result with (41) yields

$$\frac{c_l}{c_j} \le A_{lj}^{\frac{\theta \beta}{\theta + \beta}} \left(Q \max_{lj} \left(\frac{c_j}{c_l} \right) \right)^{\frac{\beta - \theta(1-\beta)}{\theta + \beta}}$$

and hence

$$max_{l,j}(\frac{c_l}{c_j}) \le (max_{l,j} A_{lj})^{\frac{\theta \beta}{\theta + \beta}} \left(Q \max_{lj} \left(\frac{c_j}{c_l} \right) \right)^{\frac{\beta - \theta(1-\beta)}{\theta + \beta}}$$

Some simple manipulations show that if

$$k \le [max_{lj}(\lambda_l/\lambda_j)]^{-\frac{\beta - \theta(1-\beta)}{2-\beta}} (max_{l,j} A_{lj})^{-\frac{\theta \beta}{2\theta - \theta \beta}}$$

then $c_l/c_j \leq 1/k$ for all $l, j \in \Omega_N$, so the NTC condition is satisfied.

Appendix B

This Appendix explains the algorithm to solve for the equilibrium of the economy in the presence of trade barriers. The case I consider is one in which there is NTNG among north countries, while South supplies all countries with global goods. I assume that $k_{Sl} = k_{lS} = k_S$, but this is purely for convenience. Given a vector x, then one can solve the system forgetting about the complementary slackness conditions by following an extension of the algorithm in Alvarez and Lucas (2005). This is as follows: first, there is a function $p_m(w)$ that solves for p_m given w. Second, there is a mapping $w' = T(w; x)$ whose fixed point, $w = F(x)$, gives the equilibrium wages given x.

The final step is to solve for the equilibrium x. Let $c(x)$ be the vector of unit costs $(c_1, c_2, ..., c_I)$ associated with x (computed from $w = F(x)$, $p_m(w)$ and $c_l = Bw_l^\beta p_{ml}^{1-\beta}$), and let $G(x) = \arg\min_{x'} x' c(x)$. Also, let $\varphi(x) = G(x)c(x)/xc(x)$ and $H_\mu(x) = \varphi(x)^\mu x + (1 - \varphi(x)^\mu)G(x)$ for $\mu \in \mathbb{N}$. Note that if $\widetilde{x}$ is a fixed point of $H_\mu(x)$ then $\widetilde{x} \in G(\widetilde{x})$, which implies that $\widetilde{x}$ satisfies the complementary slackness conditions. The algorithm to find the equilibrium x is to start with $G(0)$ and then iterate on $x' = H_\mu(x)$ until $\varphi(x)$ is sufficiently close to one. A higher value of μ makes convergence faster, but too high a value leads to oscillation and no convergence. A value $\mu = 3$ worked fine and led to $\varphi(x)$ in less than 500 iterations.

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